Summary

- Current injectors are deployed on Pegasus for helicity injection start-up and current drive
- A predictive model of the relationship between injection voltage and current is necessary for scaling this technique up
- Injector I-V characteristics scaling as $I_{INJ} \sim n_e V_{INJ}^{3/2}$ suggest a double layer governs the injector for low V_{INJ} and I_{INJ}
- At high power >100kAV, $I \sim n_e V^{1/2}$ scaling is observed
- When injectors are operated as simple electrodes, current injection occurs via cathode spots
  - Impedance remains consistent with that predicted by the Child-Langmuir Law for double sheaths
  - Area relevant to helicity injection I_p limits may no longer scale as injector area
- Initial experiments are being conducted to achieve more efficient use of area with hollow cathode-style injectors

Outline

- Motivation
- Helicity injection startup and current growth in Pegasus
- Limiting currents in intense beams
- Initial impedance measurements of Pegasus injectors
- Proposed work

Motivation

- DC helicity injection is a promising technique for tokamak startup and growth
- The simple model for injection Pegasus depends on two limits on plasma current related to the injector circuit
  - Helicity balance limit depends on injector voltage V_{INJ}:

$$I_p \le \frac{A_p}{2\pi R_0 \langle \eta \rangle}\left(V_{ind} + V_{eff}\right) \qquad V_{eff} \approx \frac{A_{inj} B_{\phi,inj}}{\Psi_T} V_{inj}$$

  - Taylor relaxation limit depends on injector current I_{INJ}:

$$I_p \le f(\varepsilon,\delta,\kappa)\sqrt{\frac{\kappa A_p I_{TF} I_{INJ}}{2\pi R_0 w}}$$

- Injector impedance fixes relationship between V_{INJ} and I_{INJ}
  - Determined by plasma conditions
  - Determines feasibility and power requirements for scaling this technique up

Helicity Balance Places a Limit on Plasma Current

Total helicity K in a tokamak geometry: $K = \int_V (\mathbf{A} + \mathbf{A}_{vac}) \cdot (\mathbf{B} - \mathbf{B}_{vac})\, d^3x$

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J} \cdot \mathbf{B}\, d^3x - 2\frac{\partial \psi}{\partial t}\Psi - 2\int_A \Phi \mathbf{B} \cdot d\mathbf{s}$$

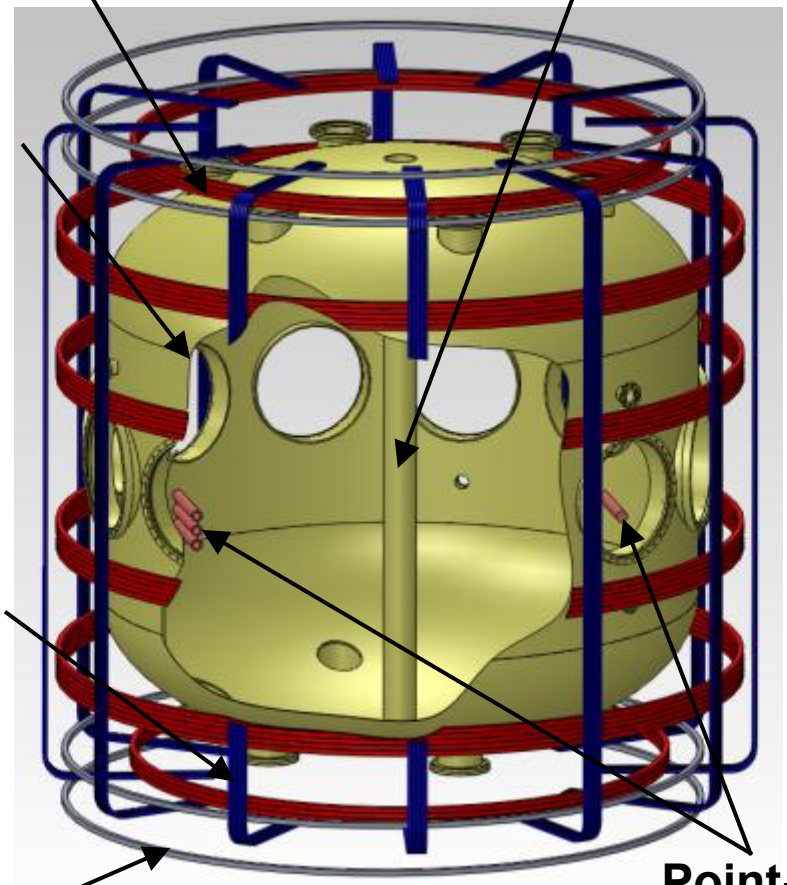
- **Resistive Helicity Dissipation**
  - $\mathbf{E} = \eta\mathbf{J} \rightarrow$ much slower than energy dissipation ($\eta\mathbf{J}^2$)
  - Turbulent relaxation processes dissipate energy and conserve helicity
- **AC Helicity Injection:** $\dot{K}_{AC} = -2\frac{\partial \psi}{\partial t}\Psi = 2V_{loop}\Psi$
  - Ψ is toroidal flux, ψ is poloidal flux (*e.g.*, current drive through solenoid induction)
- **DC Helicity Injection** $\dot{K}_{DC} = -2\int_A \Phi \mathbf{B} \cdot d\mathbf{s} = 2V_{inj}B_{\perp}A_{inj}$
  - Φ is electrostatic potential

$$I_p \le \frac{A_p}{2\pi R_0 \langle \eta \rangle}\left(V_{ind} + V_{eff}\right)$$

$$V_{eff} \approx \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{bias}$$

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Pegasus is Exploring Non-Solenoidal Startup via DC Helicity Injection



Experimental Parameters

Parameter	Achieved	Goals
A	1.15 – 1.3	1.12 – 1.3
R(m)	0.2 – 0.45	0.2 – 0.45
I_p (MA)	≤ .21	≤ 0.30
I_N (MA/m-T)	6 – 12	6 – 20
RB_t (T-m)	≤ 0.06	≤ 0.1
κ	1.4 – 3.7	1.4 – 3.7
τ_{shot} (s)	≤ 0.025	≤ 0.05
β_t (%)	≤ 25	> 40

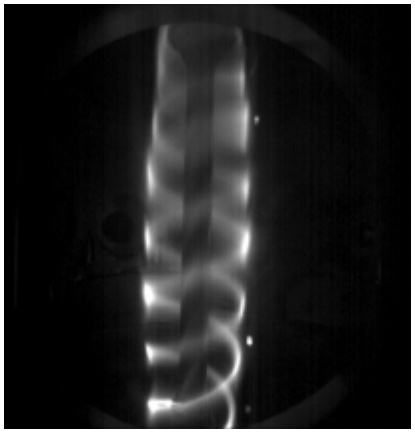
Major research thrusts include:

- *Non-inductive startup and sustainment*
- *Tokamak physics in small aspect ratio:*
  - *High-I_N, high-β operating regimes*
  - *ELM-like edge MHD activity*

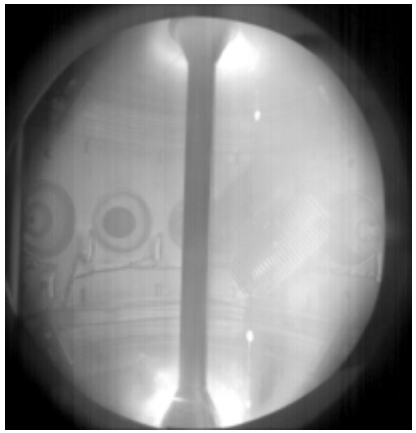
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Relaxation of Injected Current Streams Produces Tokamak-like Plasmas with Current Multiplication

- Flexible configuration options
- Inboard injection maximizes helicity input rate
- Centerstack location reduces position control requirements

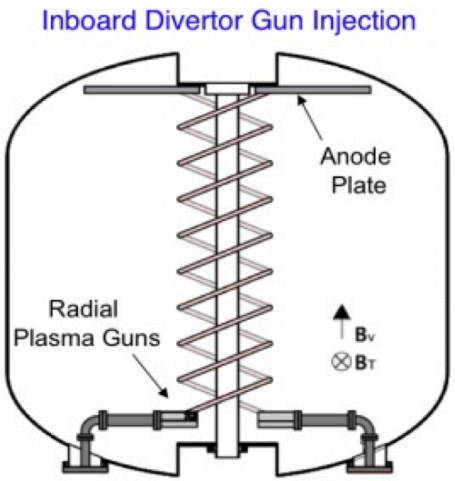


Relaxation

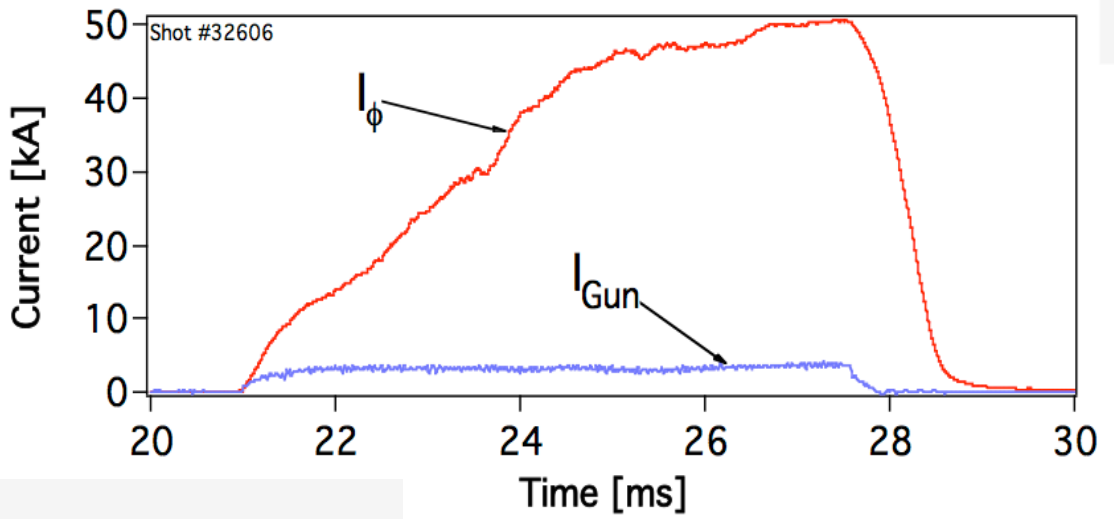


Intense Electron Beams

Tokamak-like plasma



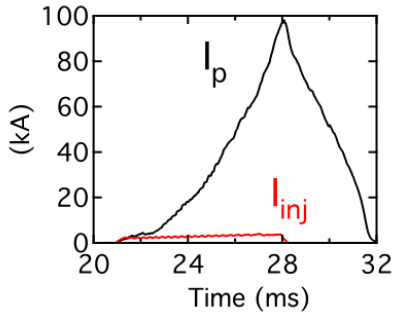
R_{gun} = 16 cm, Z_{gun} = - 75 cm



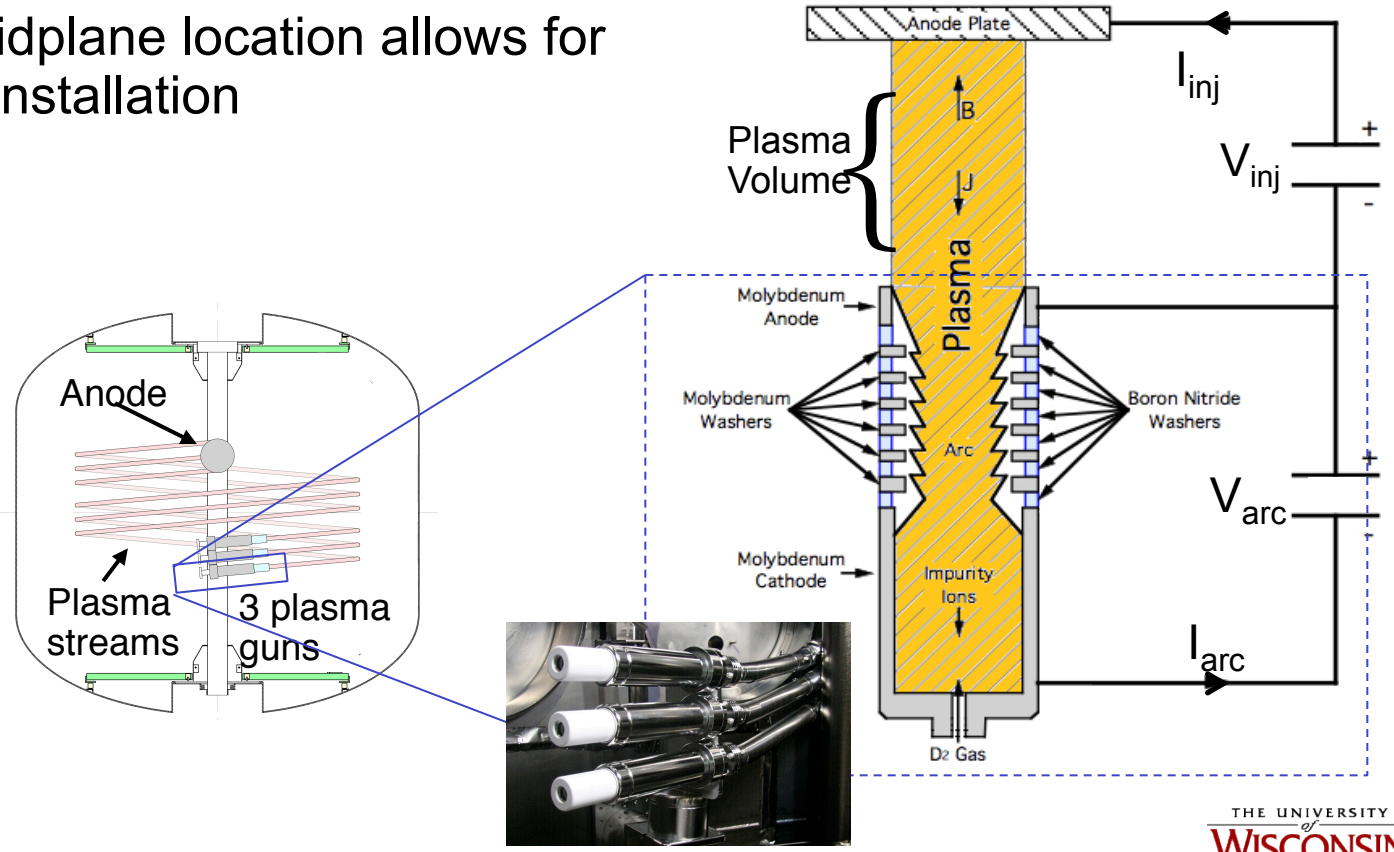
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Injectors Are Currently Mounted at Pegasus' Outboard Edge

- Outboard injection assisted by ramping PF

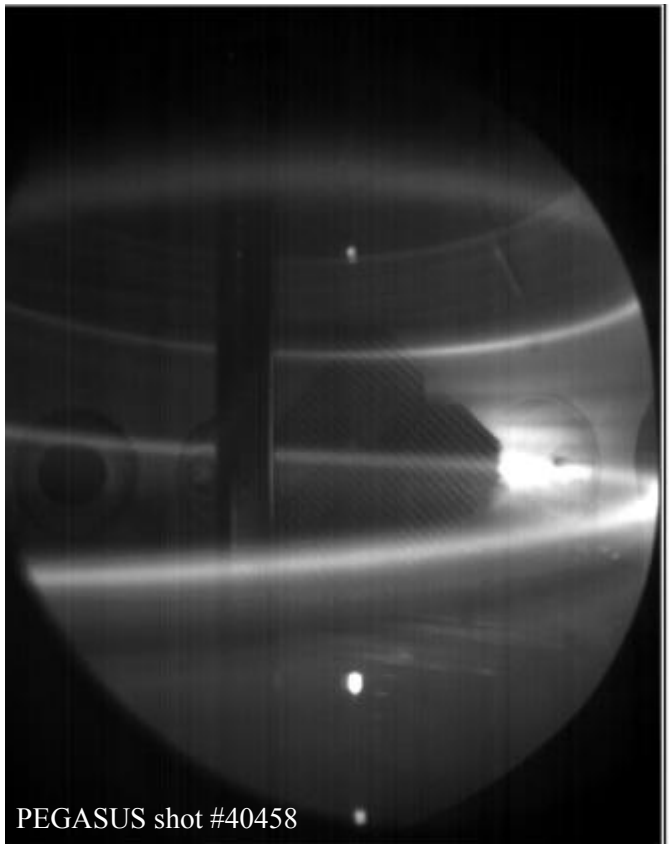


- Outboard midplane location allows for "port plug" installation

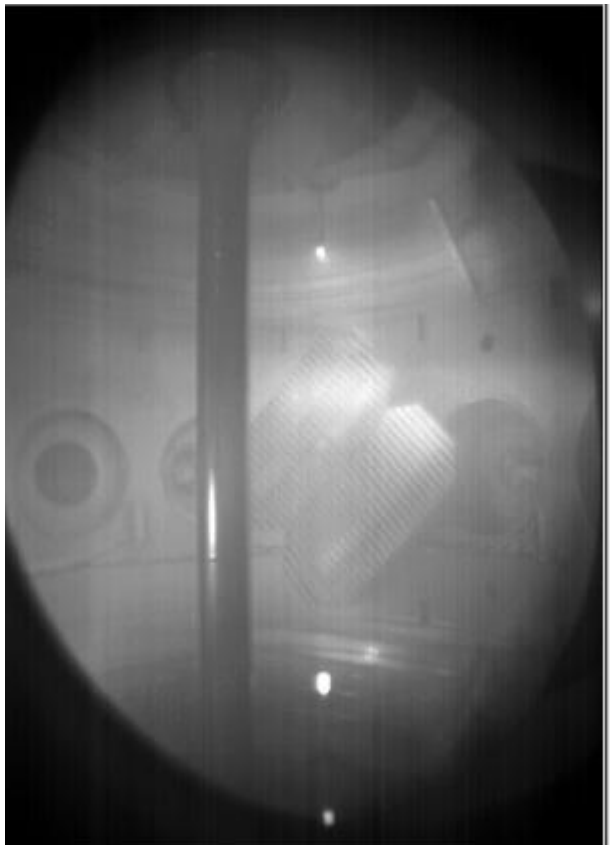


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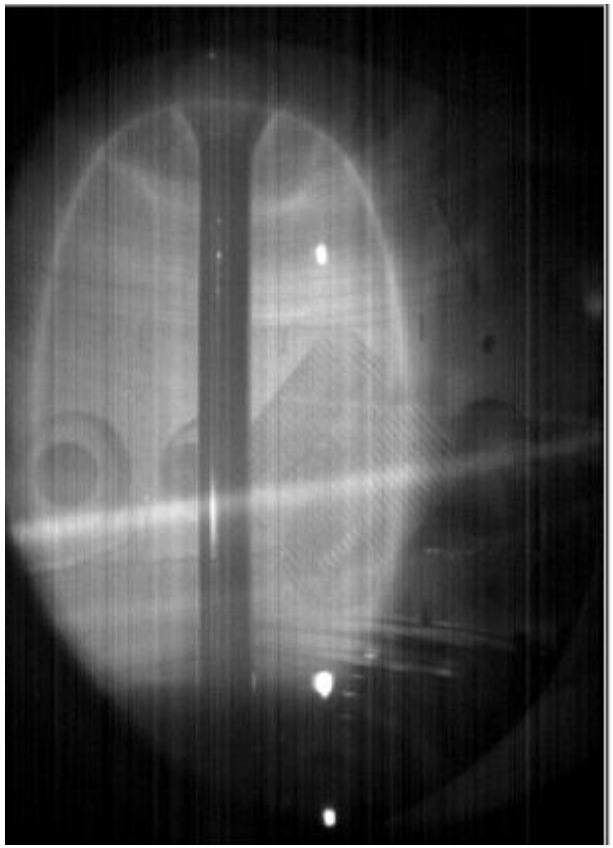
Typical Evolution Yields $I_p >> I_{INJ}$ and Apparent Tokamak-like structure



t=21.1 ms, I_p=2-3 kA
Filaments only



t=28.8 ms, I_p=42 kA
Driven diffuse plasma



t=30.6 ms, I_p=37 kA
Guns off, Decaying

See Battaglia *et al.*, Nucl. Fusion **51**, 073029 (2011)

Space-Charge Functions as Current Limit

- Charge carriers transit across biased gap without neutralization
- Particles' collective charge alters potential profile, preventing further current increase at that voltage
  - Availability of more charge carriers at the cathode does not increase current across the gap

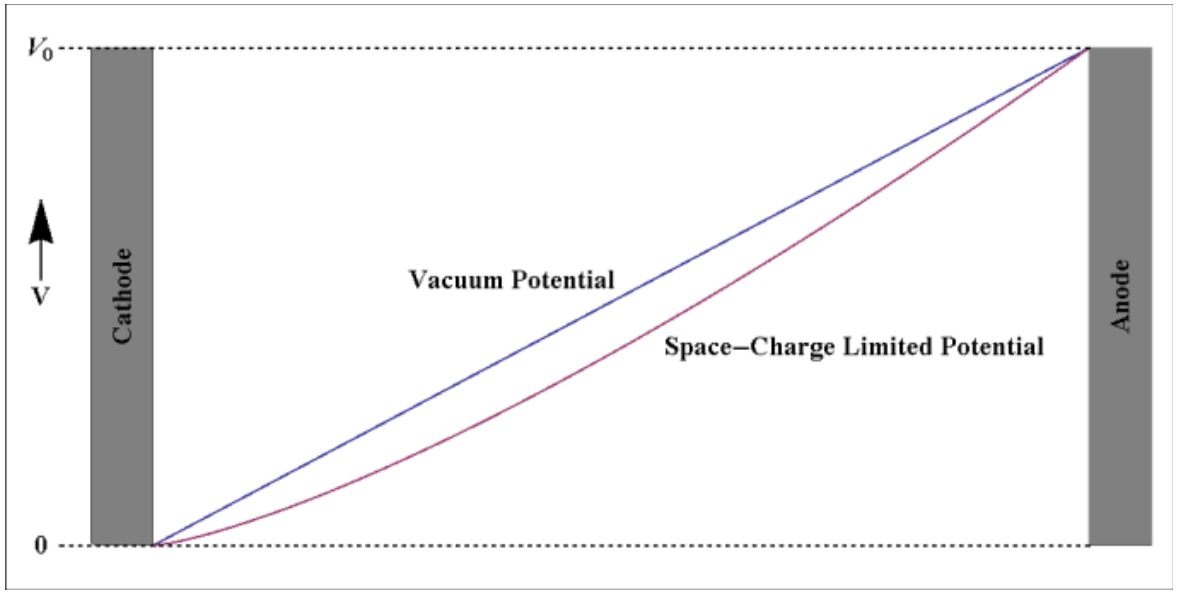
$$J_s = n_s(x) e v(x)$$

$$v(x) = \sqrt{\frac{2eV(x)}{m_s}}$$

$$-\varepsilon_0 \frac{\mathrm{d}^2 V(x)}{\mathrm{d}x^2} = n_s(x) e$$

$$\frac{\mathrm{d}^2 V(x)}{\mathrm{d}x^2} = \frac{1}{\varepsilon_0} \frac{J_s}{\sqrt{2e\dfrac{V(x)}{m_s}}}$$

$$J = \frac{4}{9}\varepsilon_0 \left(\frac{2e}{m_e}\right)^{\frac{1}{2}} \frac{V_0^{\frac{3}{2}}}{l^2}$$



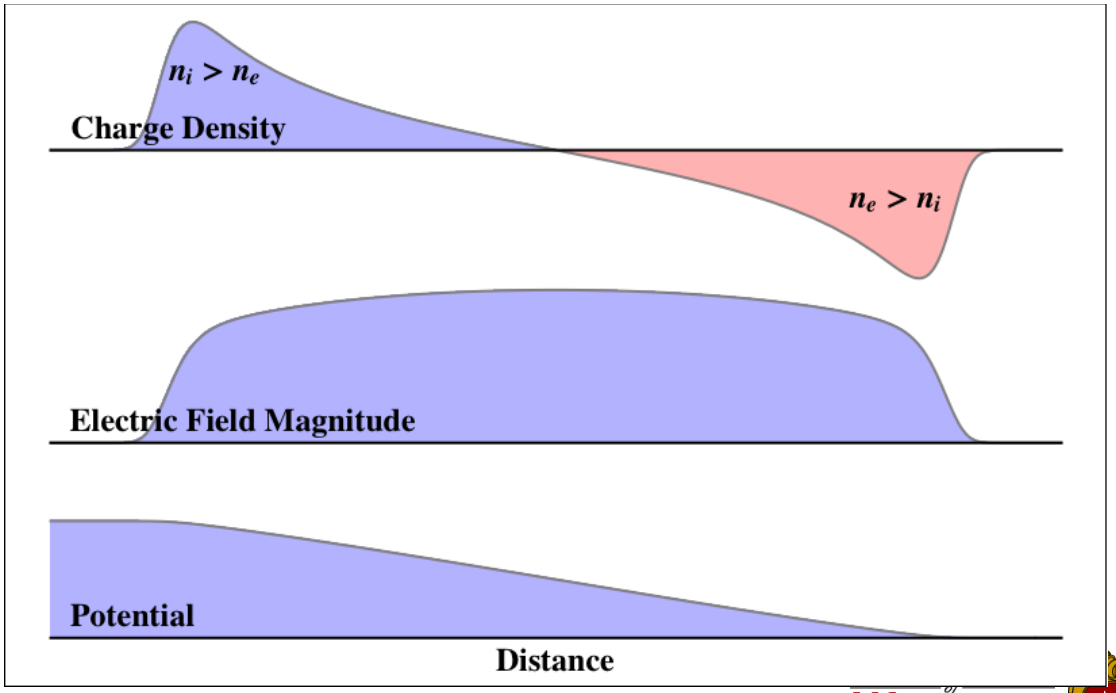
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Double Layers are Self-Organized Space-Charged Gaps in Plasmas

- Space charge gap width a plasma-determined quantity
  - Width is some number of Debye lengths
  - Smaller gap can transmit very large currents compared to typical ‘vacuum diode’ with ~cm dimensions at same voltage
- Bipolar flow of charge carriers (electrons and ions cross gap)

$$J = \frac{4}{9} 1.865 \varepsilon_0 \left(1 + \sqrt{\frac{m_e}{m_i}}\right) \left(\frac{2e}{m_e}\right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{l^2}$$

$$l^2 = \left(\lambda_{De} \cdot \chi\right)^2 = \frac{T_e \varepsilon_0}{n_e e} \chi^2$$



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Simulations Find Double Layers Expand

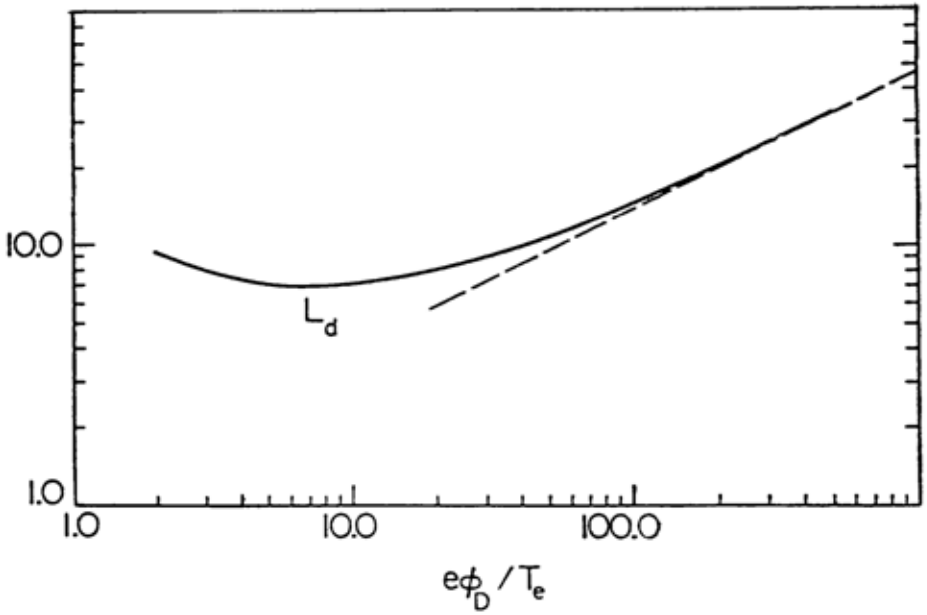
- Double layer simulation work finds increasing thickness for large potential steps, $eV_{Bias}/kT_e \geq 10$
- Thickness increases as $V^{1/2}$, (Hershkowitz 1985, Goertz 1975,1978)
- Resulting I-V relationship through layer changes to $I \sim V^{1/2}$

$$J \sim \frac{V^{\frac{3}{2}}}{l^2}$$

$$l^2 = \left(\lambda_{De} \cdot \chi\right)^2$$

$$\chi \sim V^{1/2}$$

$$J \sim \frac{V^{\frac{3}{2}}}{\chi^2} \sim \frac{V^{\frac{3}{2}}}{V} \sim V^{\frac{1}{2}}$$

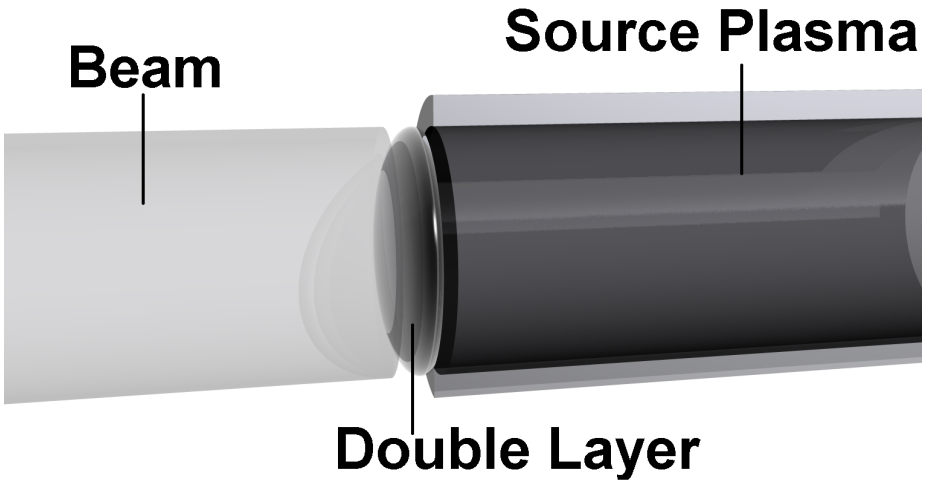


Double Layers Are Encountered in Plasma-Immersed e-Beam Sources

- Space charge gap width several Debye lengths
  - $\lambda_{De}^{tok.}\left(10\ \mathrm{eV}, 10^{18}\ \mathrm{m}^{-3}\right) = 20\,\mu\mathrm{m}, \quad \lambda_{De}^{arc}\left(10\ \mathrm{eV}, 10^{21}\ \mathrm{m}^{-3}\right) = 0.7\,\mu\mathrm{m}$
  - Small gap produces very large currents at low voltages compared with cm scale vacuum gaps
  - Operate as very efficient particle accelerators

$$J = \frac{4}{9} 1.865 \varepsilon_0 \left(1 + \sqrt{\frac{m_e}{m_i}}\right) \left(\frac{2e}{m_e}\right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{l^2}$$

$$l^2 = \left(\lambda_{De} \cdot \chi\right)^2 = \frac{T_e \varepsilon_0}{n_e e} \chi^2$$



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Current Sources in Pegasus are Intense Beams

- Space charge limit resulting from double layer is very low impedance
  - $I = 1 \times V^{3/2}$ implies current gets very 'large' for 'small' V - typical ramps achieve J~0.5kA/cm^2 and I_{Bias}~1kA at 100V
  - Self-field of beam is of the order of the local toroidal field

- As current increases, self-magnetic field becomes more restrictive limit than self-electric field due to space-charge

- Beam is subject to magnetic limiting

At Higher Currents Magnetic Limiting is Expected

- Intense beams can be defined in several ways:
  - Energy conservation:

$$W_{Kinetic}^{Beam} = W_{Magnetic}^{Beam}$$

  - Electron Larmor radius in beam field ≈ beam radius

$$r_{L,e} = \frac{r_{beam}}{2}$$

  - Solving either equation for I defines Alfven current

$$I_A = \frac{4\pi m v}{e\mu_0} = \frac{4\pi m_e}{e\mu_0}\sqrt{\frac{2eV_{Bias}}{m_e}}$$

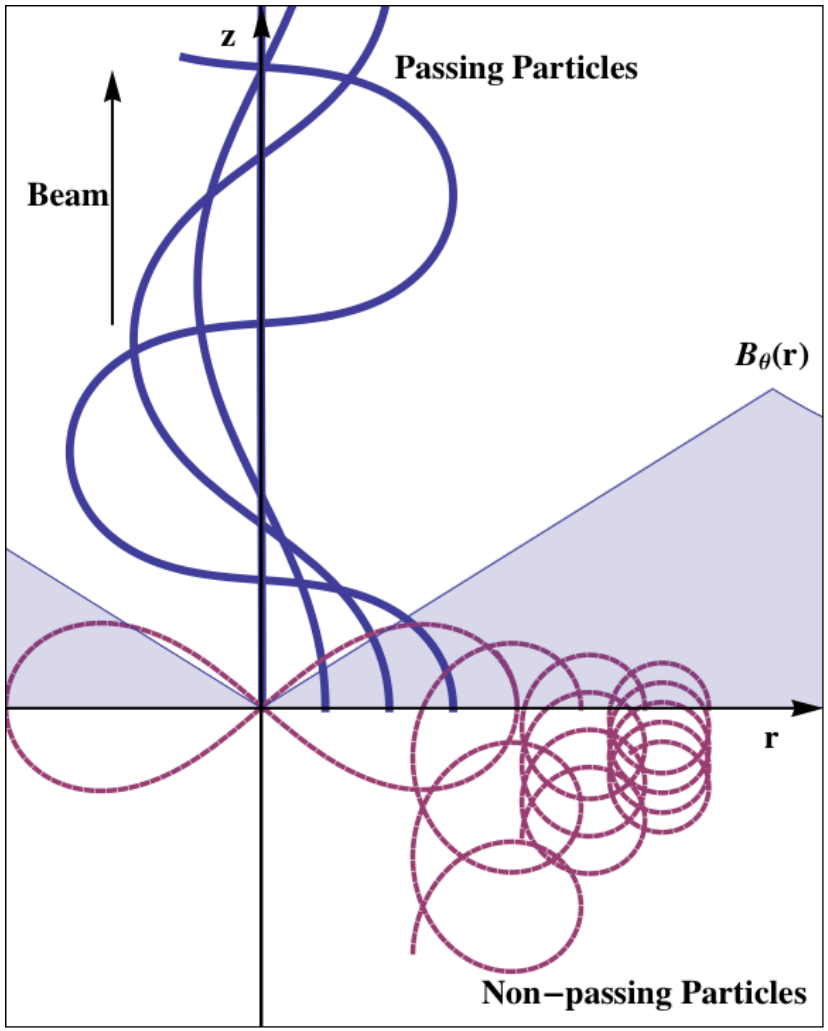
- I_A is a 'soft' limit on current

- Pegasus regularly achieves $I_{Bias} \sim 3I_A$

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Alfven-Lawson Current Limit is First Formulation of Magnetic Current Limiting

- Alfven-Lawson Current Limit
  - First posited by Alfven in 1939
  - Particles reverse direction in beam self-field
  - $I_{A-L} = 1.65 I_A = 56\sqrt{V_{Bias}}$



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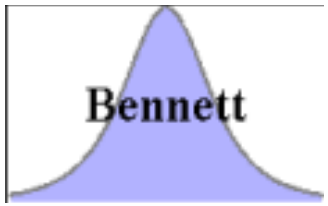
Energy Conservation Constitutes Another Form of Current Limiting

- Energy conservation, $W_{Kinetic} = W_{Magnetic}$
  - Limit depends on current profile
  - Hollower profiles have higher limits

$$f\left(\rho = \frac{r}{R}\right) \equiv \frac{I(r)}{I(R)}$$

$$I_{Limit} = \frac{4\pi}{q\mu_0} \frac{< K >}{v_{\text{drift}}} \frac{1}{\int_0^1 \mathrm{d}\rho \frac{f^2(\rho)}{\rho}}$$

$$< K >= \overline{\left(\frac{1}{2} m v^2\right)}$$



$$I_{Bennett} = 2.6 I_A = 88\sqrt{V_{Bias}}$$

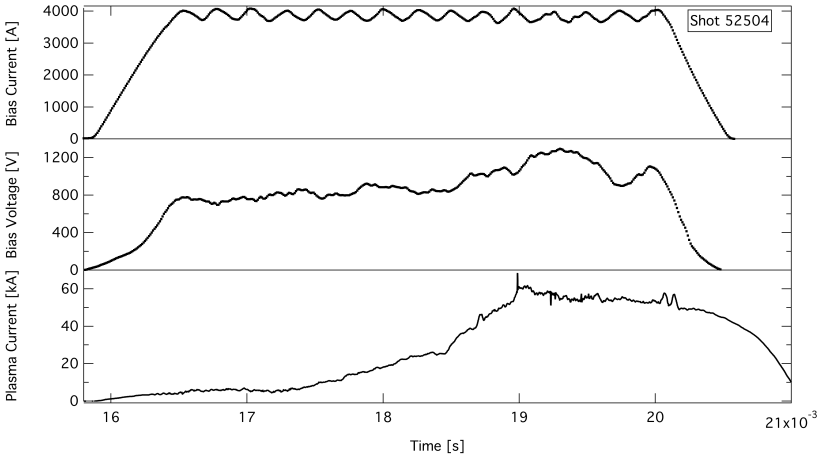


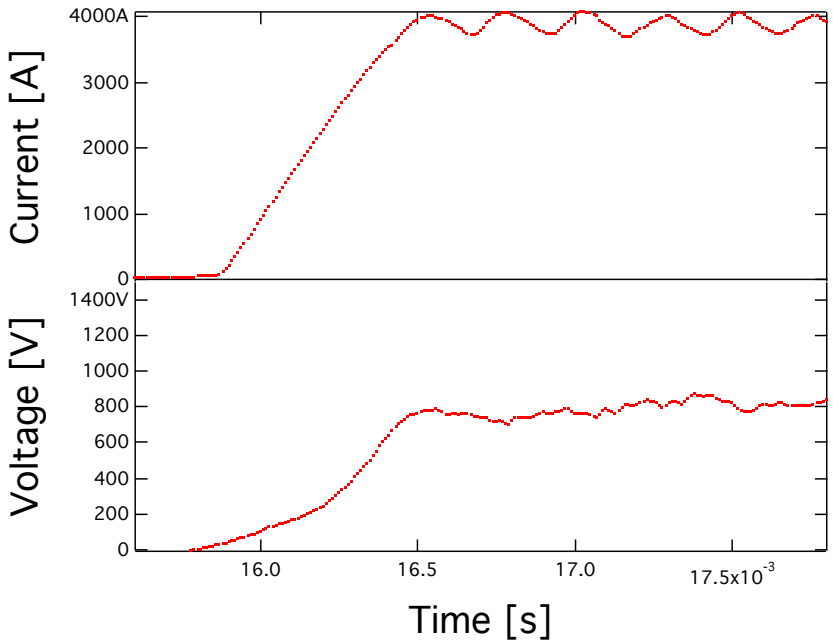
$$I_{Uniform} = 4 I_A = 130\sqrt{V_{Bias}}$$

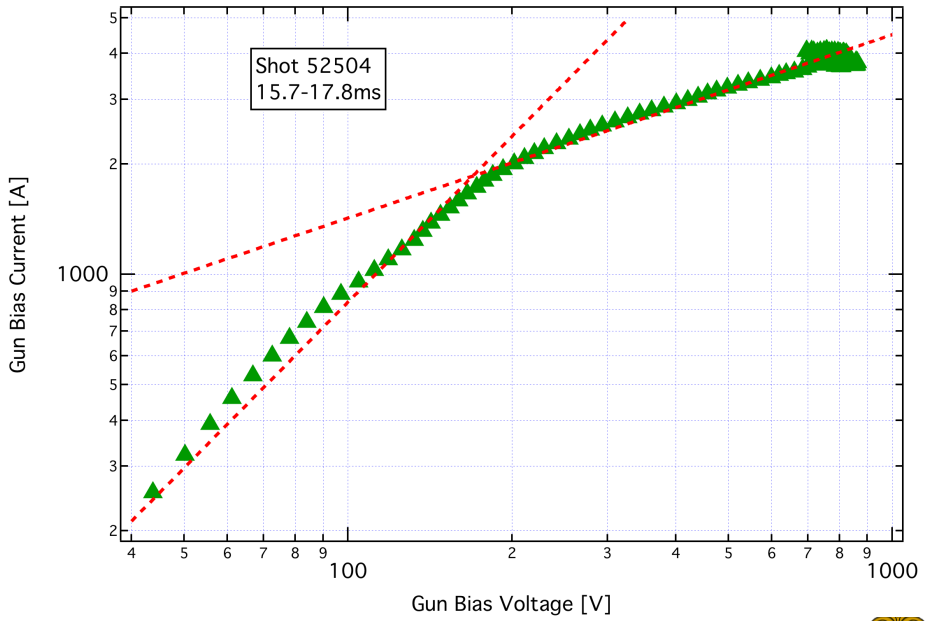
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Pegasus injectors for fixed fueling show two regimes

- Fast ramps (<1ms) to operating point reveal I-V relationship
- Two clear regimes in data
  - $I \sim V^{3/2}$ regime at low voltage
  - $I \sim V^{1/2}$ regime at higher current, voltage



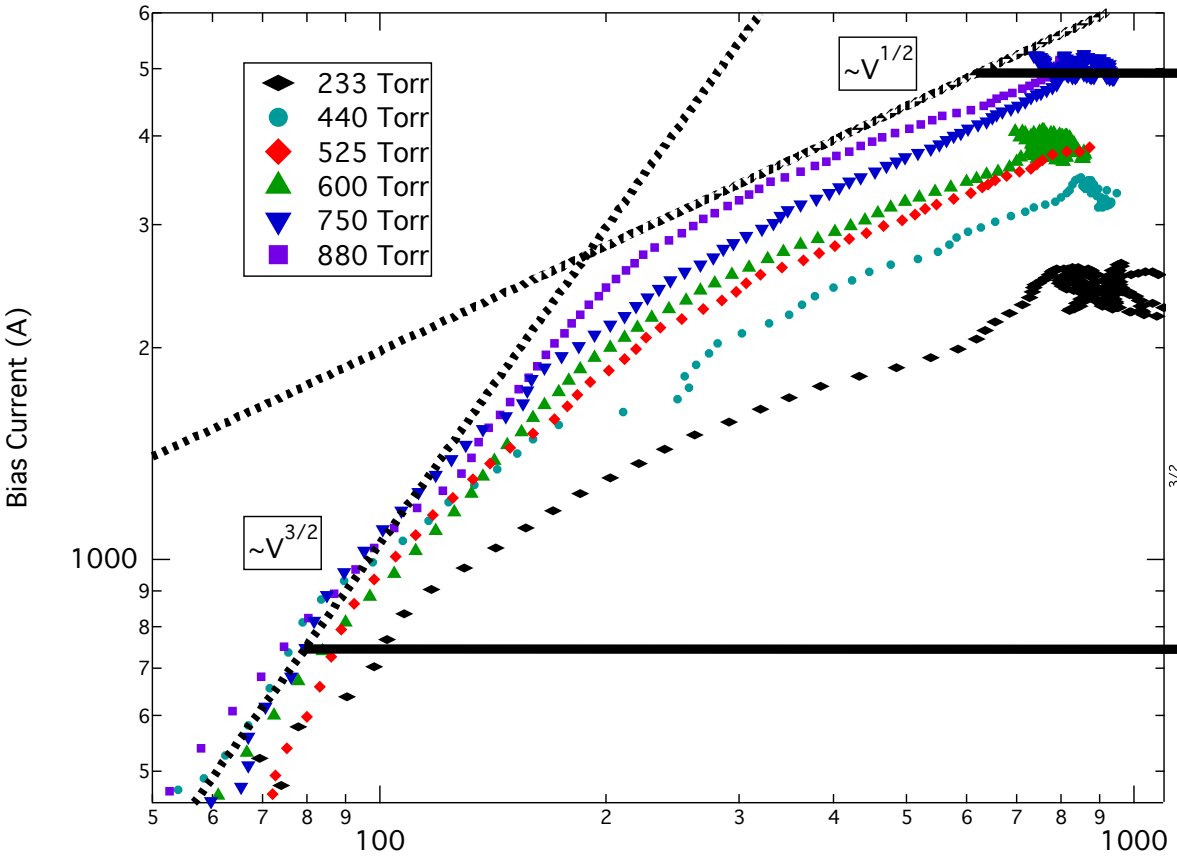


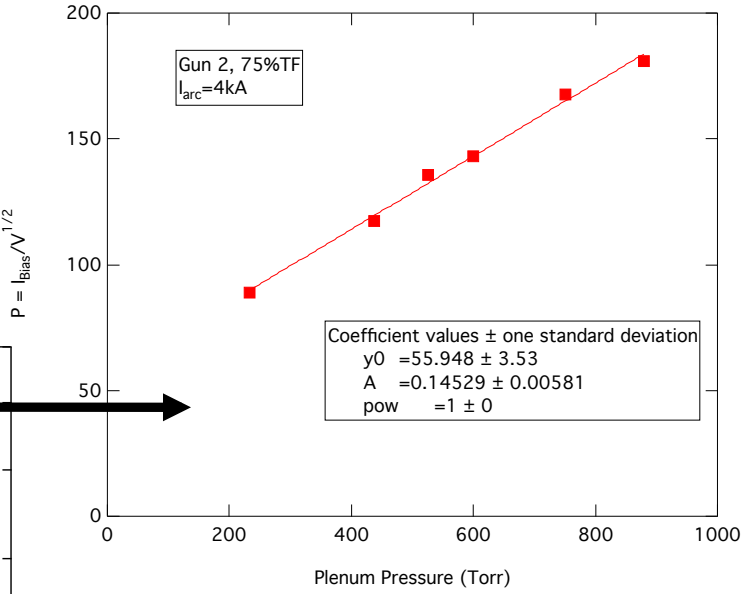


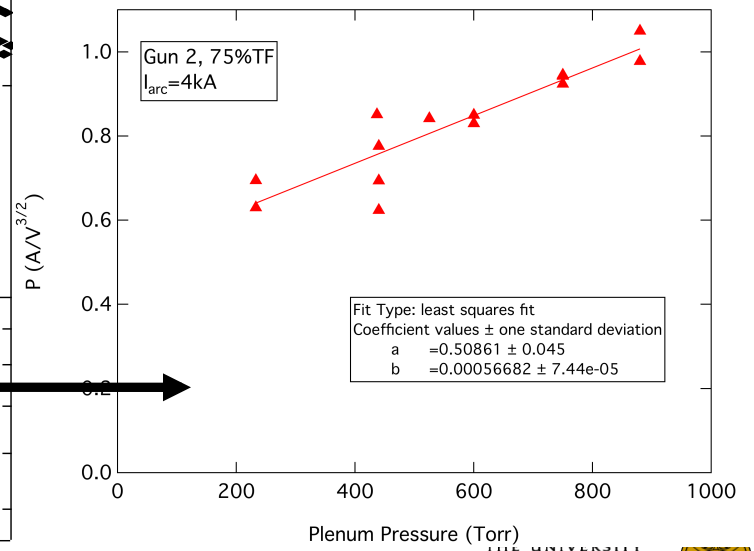
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Current increases in offset-linear fashion with fueling

- Both regimes depend on gun fueling in offset-linear fashion
- Note: Shots include prefill fueling



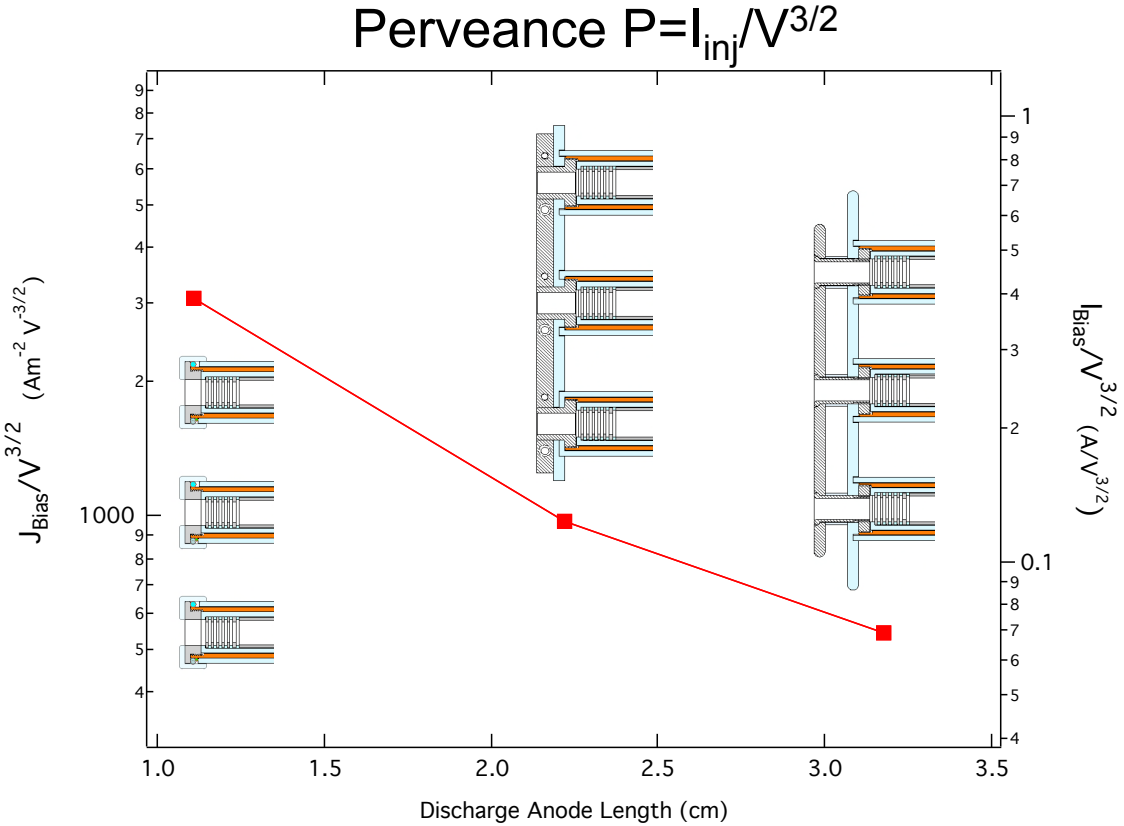




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Perveance seen to decrease as discharge anode length increased

- Increasing perveance with discharge anode length consistent with increasing distance between double layer and arc plasma source
- Injector breakdown observed at lower voltages



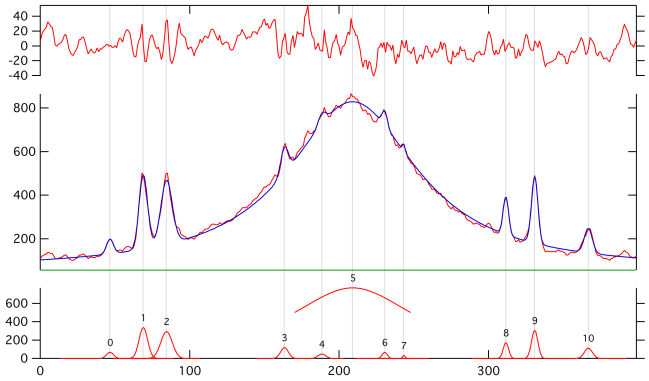
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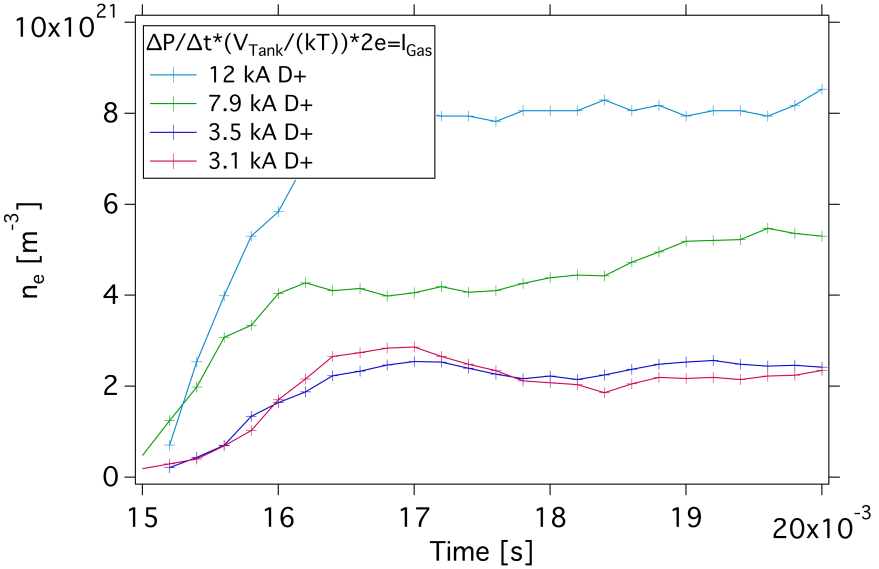
Stark Broadening of H_δ Shows Arc Source Plasma Density is Linear in Fueling

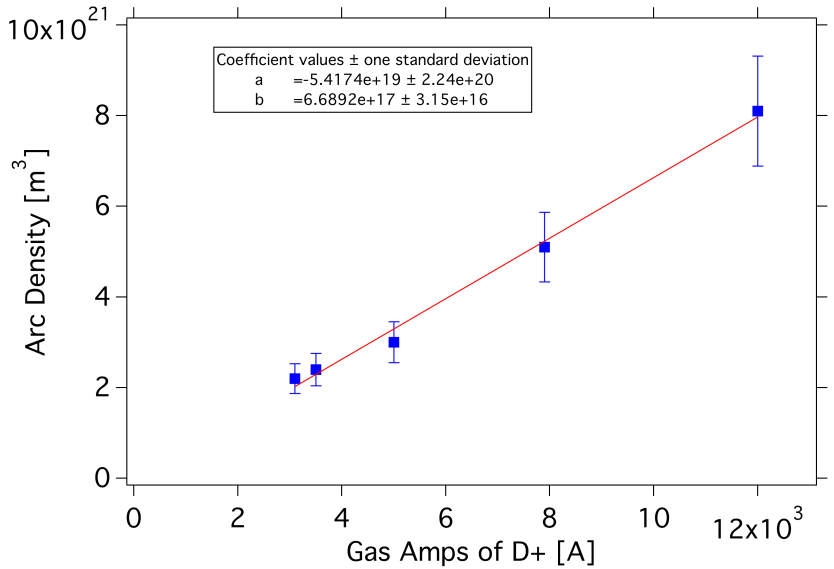
- H_δ profile obtained with view down center of gun
- Peak arc source plasma density is order $10^{21} m^{-3}$ for Pegasus' operating space

$$\Delta\lambda_{H-\delta}^{FWHM} = 0.92\left(n_e^{20}\right)^{2/3}[\text{A}]$$

$$n_e^{20} \approx 10 \Rightarrow \Delta\lambda_{H-\delta}^{FWHM} \sim 4 \text{ A}$$



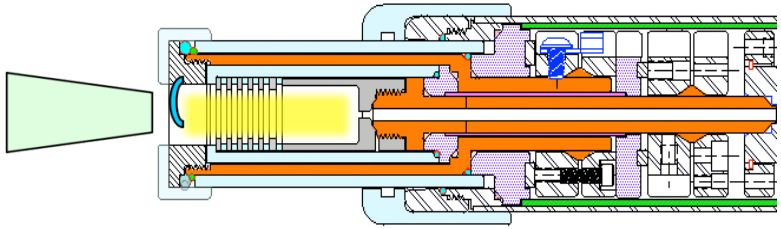




Current Through Double Layer is Predicted to scale as ~$V^{3/2}$ and ~n_e

- Space-charge limited current through double-layer
  - depends on plasma parameters (n_e, T_e)
  - is proportional to current injection area

$$J = \frac{4}{9} 1.865 \varepsilon_0 \left(1 + \sqrt{\frac{m_e}{m_i}}\right) \left(\frac{2e}{m_e}\right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{l^2}$$



Where l is the sheath thickness and is some number of λ_{De}

$$l^2 = \left(\lambda_{De} \cdot \chi\right)^2 = \frac{T_e \varepsilon_0}{n_e e} \chi^2$$

$$J_{inj} \sim \frac{V^{\frac{3}{2}}}{l^2} \sim \frac{V^{\frac{3}{2}}}{T_e} \frac{n_e^{DL}}{\chi^2}$$

$$I_{inj} \sim \frac{V^{\frac{3}{2}}}{T_e} \frac{n_e^{DL}}{\chi^2} A_{inj}$$

$$n_e \sim 10^{20}\ \mathrm{m}^{-3}$$
$$A = 2\ \mathrm{cm}^2 \quad \Rightarrow \quad I_{inj} \approx 1 \cdot V^{\frac{3}{2}}$$
$$T_e \sim 10\ \mathrm{eV}$$

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Double layer concept also offers framework to understand I~$V^{1/2}$ regime

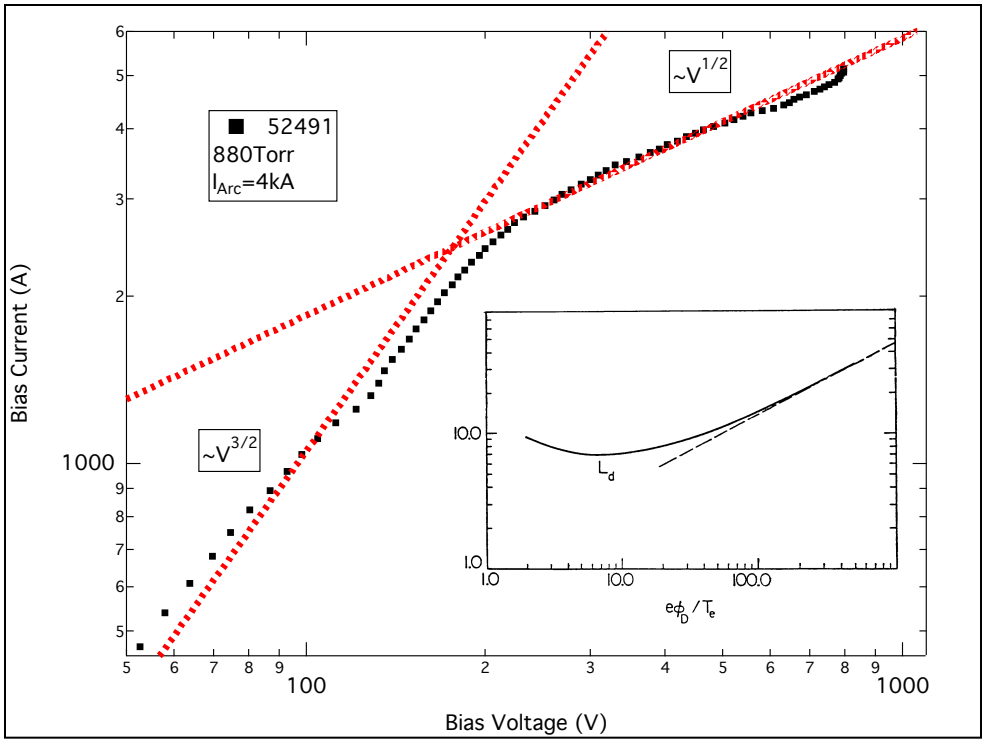
- MST has estimated electron temperature at ~10 eV
- Half-power regime occurs at 100-200 V, consistent with expectation ($eV_{Bias}/kT_e \geq 10$)

$$J \sim \frac{V^{\frac{3}{2}}}{l^2}$$

$$l^2 = \left(\lambda_{De} \cdot \chi\right)^2$$

$$\chi \sim V^{1/2}$$

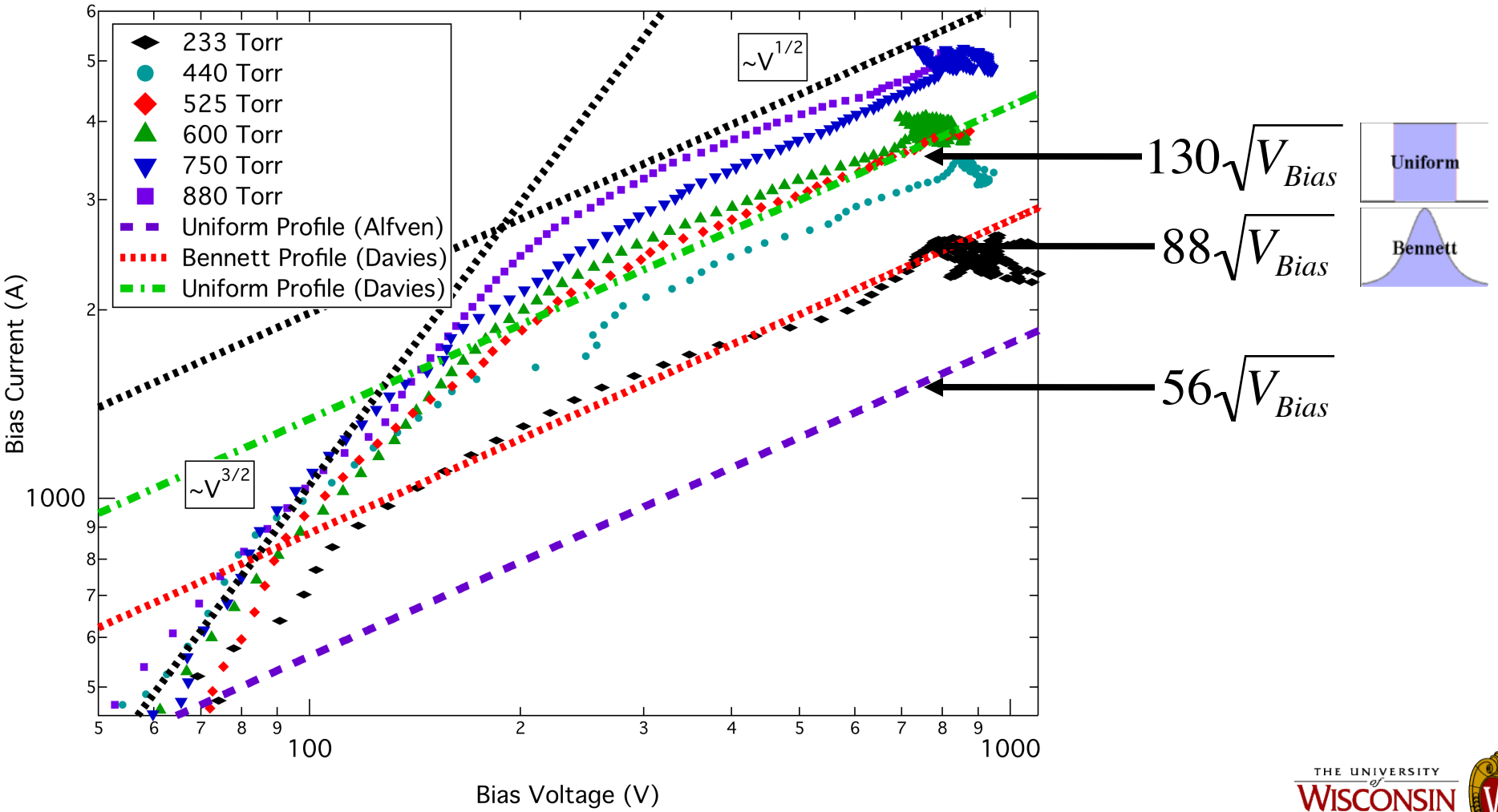
$$J \sim \frac{V^{\frac{3}{2}}}{\chi^2} \sim \frac{V^{\frac{3}{2}}}{V} \sim V^{\frac{1}{2}}$$



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Magnitude, scaling of data also consistent with magnetic limits

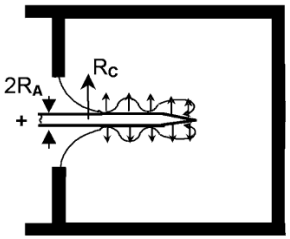
- Magnetic limit is expected at currents of order I_A
- Would suggest current profile is changing with density

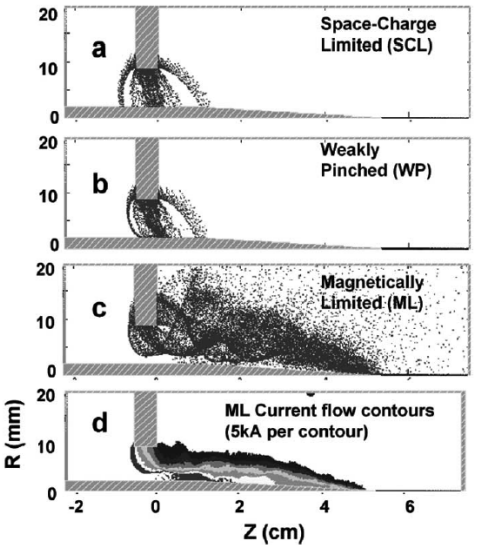


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Similar Overall I-V Behavior Seen Relativistic Magnetically-Limited Diodes

- Example: Rod pinch diode can be driven to from SCL to magnetically-limited regime
  - Current magnitude at high current in the non-relativistic limit is order I_A and scales as $V^{1/2}$





J. Maenchen. Proc. IEEE. 92 1021 2004

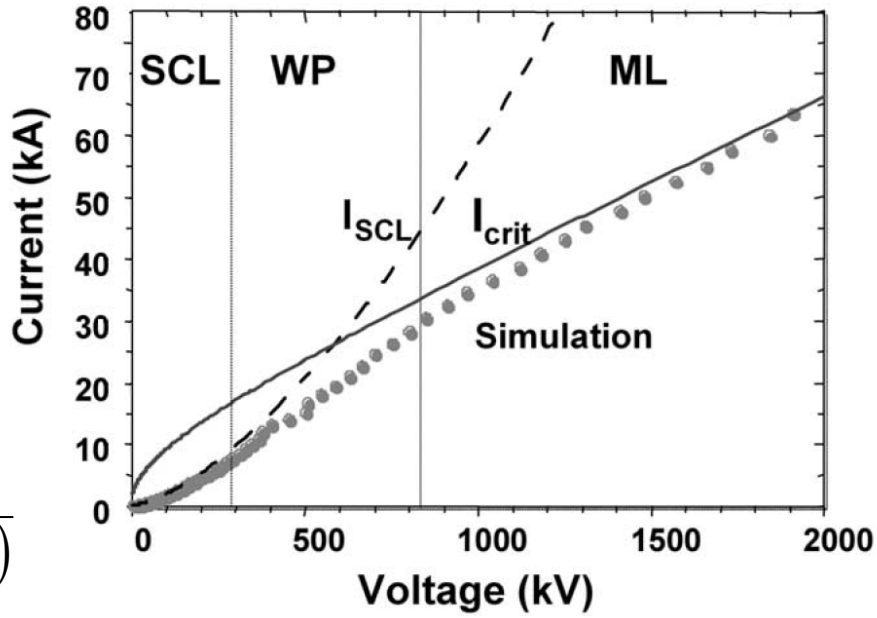
$$I_{Mag}^{RP} \sim I_A(v=c)\frac{\left(\gamma^2-1\right)^{1/2}}{\ln\left(R_C/R_A\right)}$$

$$\left(\gamma^2-1\right)^{1/2} = \left(\left(1-\left(\frac{v}{c}\right)^2\right)^{-1/2}-1\right)^{1/2}$$

$$\xrightarrow{v<<c} \cong \left(1+\frac{1}{2}\left(\frac{v}{c}\right)^2-1\right)^{1/2} = \frac{v}{\sqrt{2}c}$$

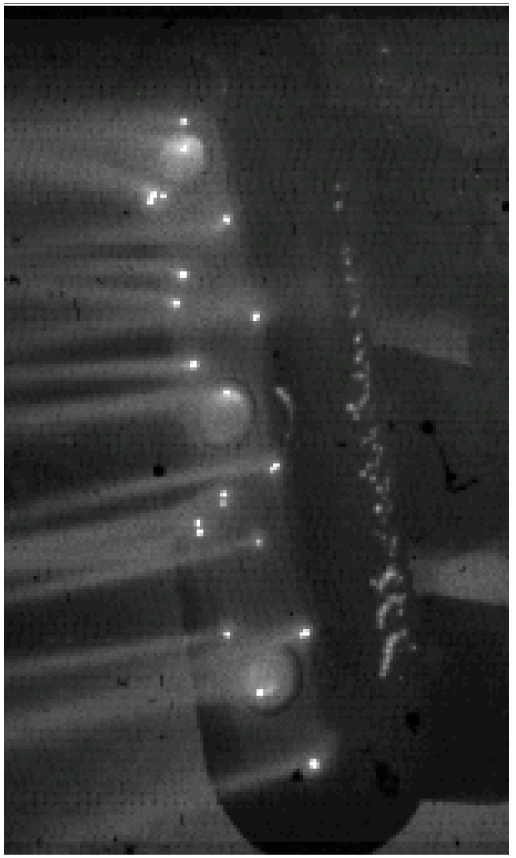
$$I_{Mag}^{Lim} \sim I_A(v=c)\frac{v}{\sqrt{2}c}\frac{1}{\ln\left(R_C/R_A\right)}$$

$$I_{Mag}^{Lim} \sim \left(\frac{4\pi mc}{e\mu_0}\right)\frac{v}{\sqrt{2}c}\frac{1}{\ln\left(R_C/R_A\right)} \sim I_A\frac{1}{\sqrt{2}\ln\left(R_C/R_A\right)}$$

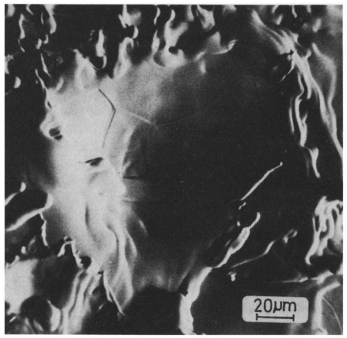


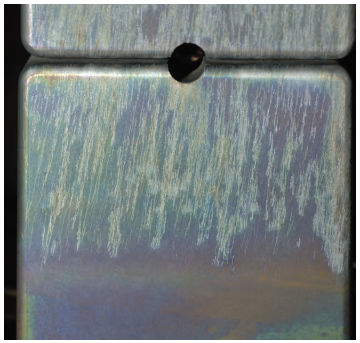
Electrode Current Injection Accomplished via Cathode Spots

- Cathode spots are known to sustain double layers
  - Measured voltage scalings support double layer hypothesis
  - Cathode spots themselves drop 10s of volts
- Localized nature suggests unfavorable area utilization
  - May be useful if effective area of injection can be spread out









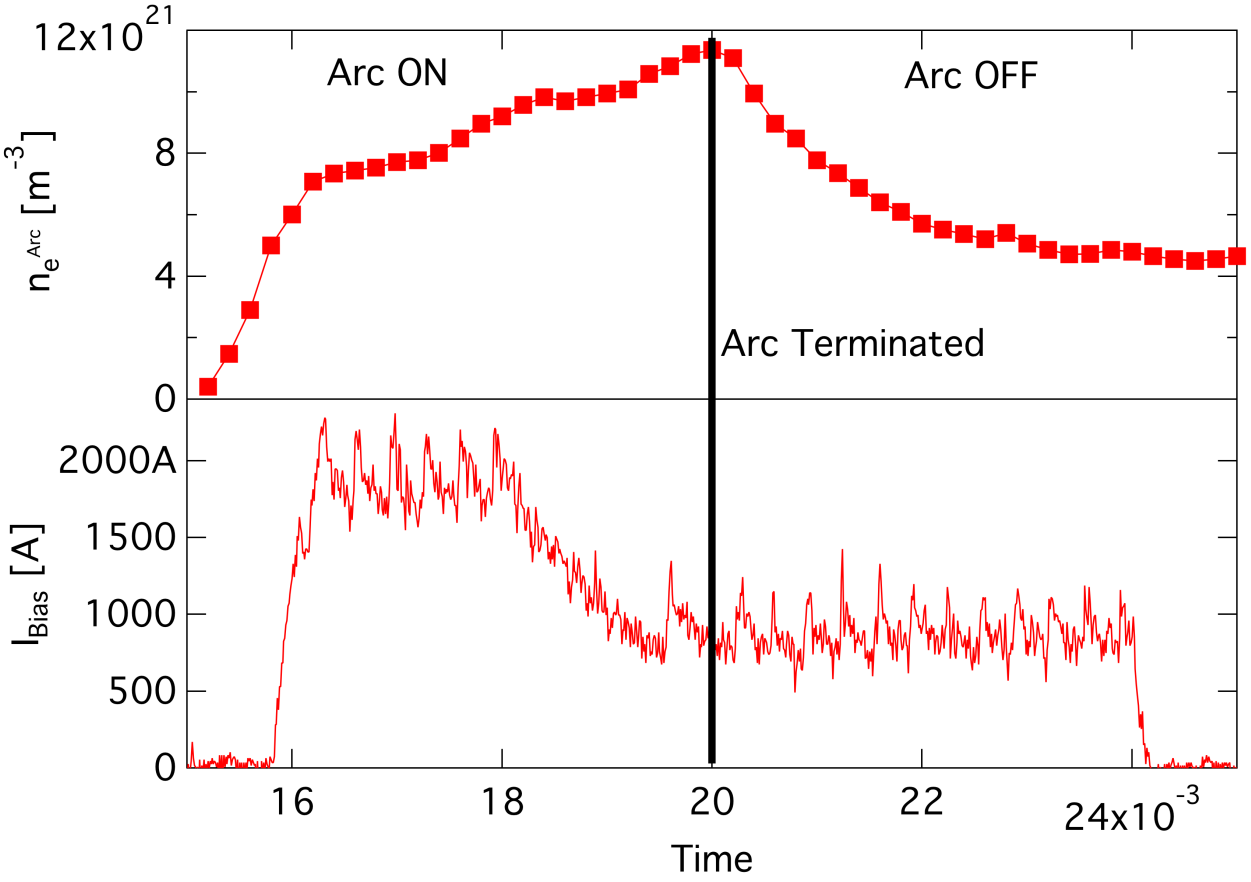
Hollow Cathode-Style Current Injection Potentially Enables Efficient Area Utilization

- Hollow cathode geometry provides high J without use of arc source plasma
  - Effective area of injection likely to correspond to geometric injector area
  - Current densities of 0.5kA/cm^2 apparently achieved
  - At onset of cathode spots, HC density crashes

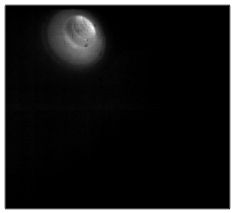
- Successful operation of 'arcless' HC suggests path forward for efficient area utilization without 100kW arc discharge
  - A successful 'hollow cathode' injection geometry would eliminate reliance on injection via cathode spots
  - Initial data suggests moderately lower current densities than arc plasma
  - Potential large-array implementation would be much simpler than arc

H_δ Broadening Arc Plasma Density Measurements Support Hollow Cathode Injection Concept

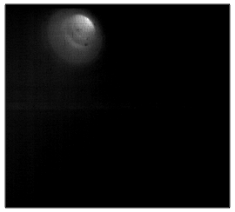
- Continuing gas injection after arc appears to suppress cathode spots for up to ~1kA bias current from injector
- Density measurements continue to indicate high 'arc'



Arc+Gas+Bias:



Gas+Bias:



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Summary

- Plasma current limits of DC helicity injection depend on injector voltage and current
  - Relationship between voltage and current is a plasma-determined quantity
  - Power supply requirements will be determined by impedance
  - Scaling of voltage/current requirements determine feasibility of technique on larger devices
- Pegasus injectors are well-described with a conceptual framework borrowed from the physics of intense beams
  - Relationship between voltage and current is determined by space-charge effects at low voltages and currents
  - At higher currents/voltages, self-magnetic field effects appear to set the injector impedance
- Hollow cathode injectors appear a promising, simpler, next step in injector design

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