Power Balance Modeling and Validation for ST Startup Using Local Helicity Injection

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PEGASUS
Toroidal Experiment

Developing Predictive Capability for Local Helicity Injection (LHI) Startup

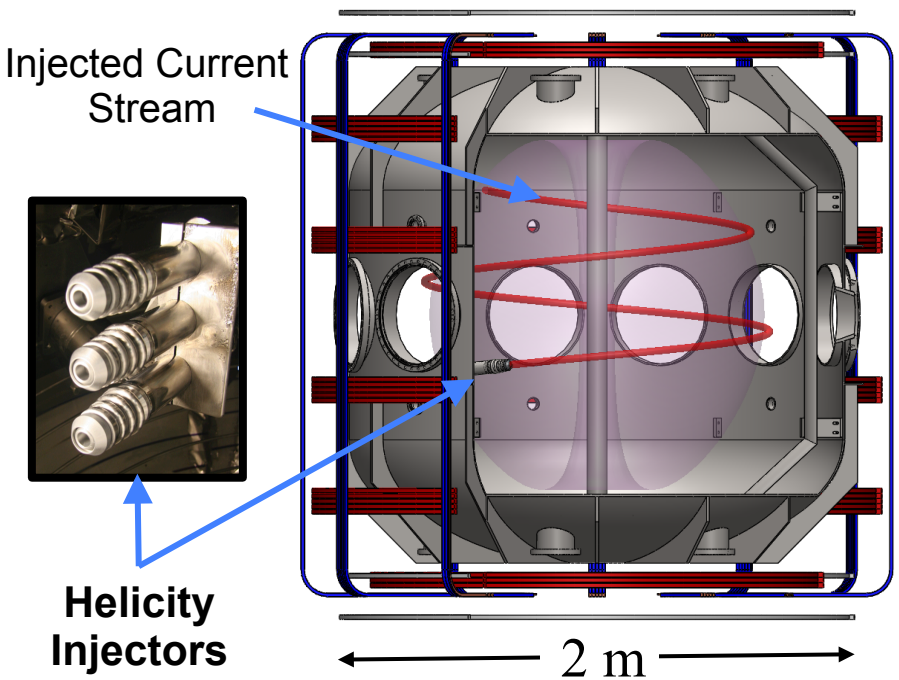
- LHI is a promising non-solenoidal startup technique
- Previous work: I_p limits from Taylor relaxation, helicity balance
- Time-evolving model needed to interpret, predict LHI $I_p(t)$
- 0-D power-balance model is developed to address this need
- The model suggests ~1 MA startup in NSTX-U startup feasible
  - Understanding transport, confinement scalings key for predictability

*Work supported by US DOE Grants DE-FG02-96ER54375 and DE-SC0006928

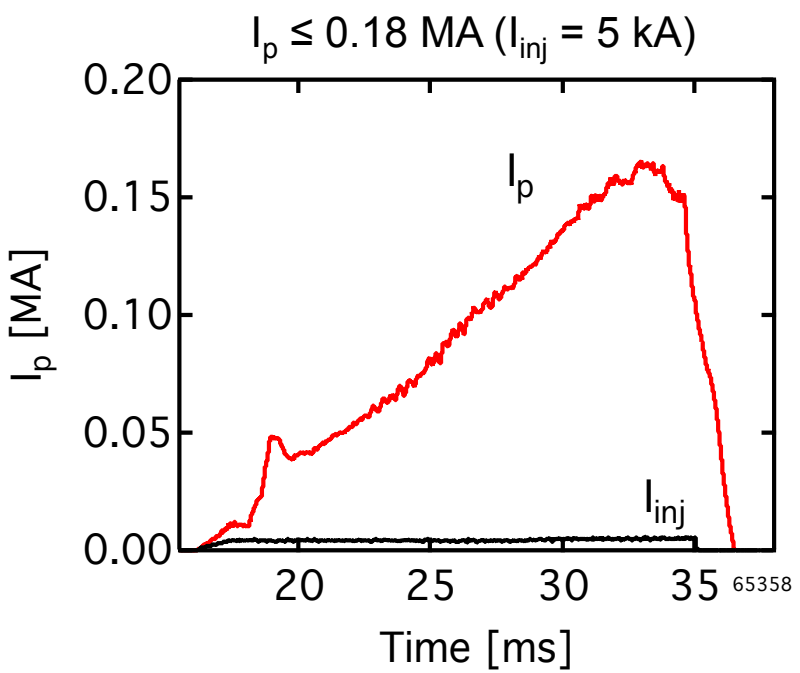
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LHI is a Scalable Non-Solenoidal Startup Technique

Edge current injected from local plasma sources



High I_p Tokamak Plasmas Produced



- Unstable current streams form tokamak-like state via Taylor relaxation
- Compact, modular, and appears scalable to MA-class startup

Global Helicity Balance, Taylor Relaxation Limit Maximum I_p

Helicity balance in a Tokamak geometry:

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J}\cdot\mathbf{B}\, d^3x - 2\frac{\partial\psi}{\partial t}\Psi - 2\int_A \Phi\mathbf{B}\cdot d\mathbf{s} \quad \Rightarrow \quad I_p \le \frac{A_p}{2\pi R_0\langle\eta\rangle}\left(V_{ind} + V_{eff}\right)$$

- Helicity injection provides an effective loop voltage:

$$V_{LHI} \approx \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{inj}$$

Taylor relaxation of a force-free equilibrium:

$$\nabla \times B = \mu_0 J = \lambda B$$

$$\lambda_p \le \lambda_{edge} \quad \Rightarrow \quad I_p \le \sqrt{\frac{1}{B_{\theta+V,inj}/I_p}\frac{\Psi_T I_{inj}}{2\pi R_{inj} w_{inj}}}$$

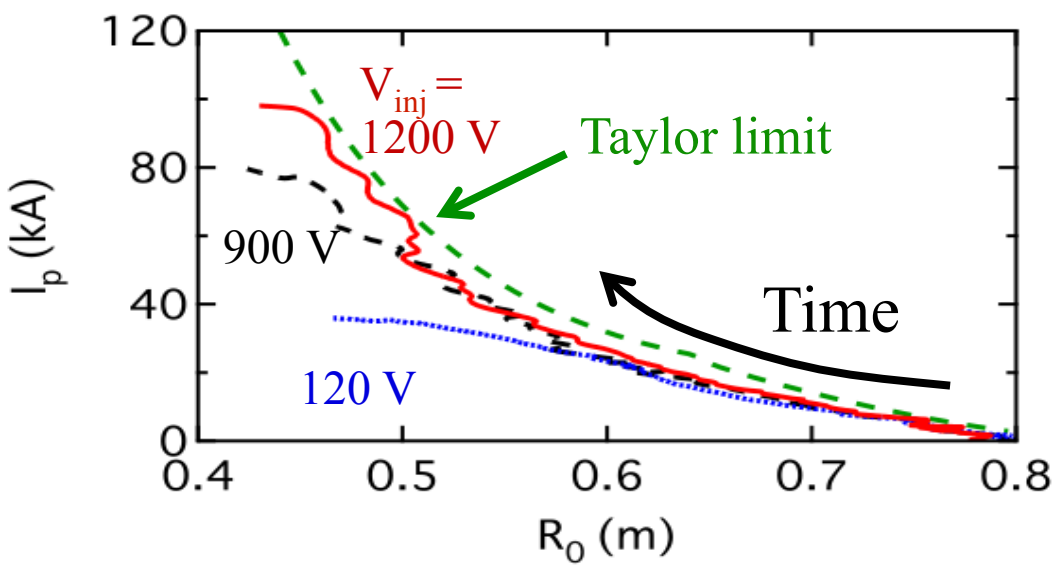
A_p	Plasma area	Ψ_T	Plasma toroidal flux
w_{inj}	Edge current channel width	$B_{\theta+V,inj}$	Poloidal field in injection region

*Eiditis et al., Journal of Fusion Energy 26, pp. 43 (2007)
Battaglia *et al.*, Nucl. Fusion **51, 073029 (2011)

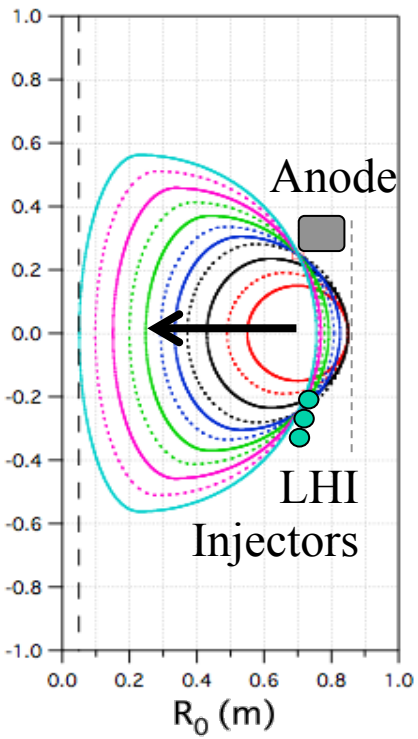
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$I_p(t)$ Predictions Must Incorporate Injector, Plasma Dynamics

- Taylor Limit is dynamic in time
  - Accessed with sufficient drive
- Understanding current drive requires accounting for dynamic plasma geometry evolution
  - Diverge from Taylor Limit trajectory with varied V_{LHI}



Estimated plasma evolution:



*Battaglia *et al.*, Nucl. Fusion **51**, 073029 (2011)



A Power-Balance Model for LHI Startup

Power-Balance, Taylor Relaxation Applied to Predict LHI $I_p(t)$

0-D LHI model predicts $I_p(t)$ based on lowest of two limits:

- Poynting's Theorem at plasma boundary sets $I_p(t)$:

$$\underbrace{I_p V_s}_{\text{Plasma surface-voltage}} \approx \underbrace{\iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right) dV}_{\text{Internal magnetic energy storage}} + \underbrace{I_p^2 R_p}_{\text{Resistive Dissipation}} - \underbrace{I_p V_{LHI}}_{\text{Non-inductive current drive (LHI)}}$$

- Taylor relaxation limit strictly enforced as maximum I_p

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Power Balance Model Incorporates Analytic Plasma Inductance Formulae

$$I_p\left[V_{PF}+V_{geo}-V_{W_m}-V_{IR}+V_{LHI}\right]=0$$

V_{IND}

Analytic low-A descriptions of L_p*, B_z**

$$V_{PF}=-\sum_{coils}\frac{d}{dt}\left[\psi_{PF}\right]\approx-\frac{\partial}{\partial t}\left[M_V\pi R_0^2\,B_V\big|_{R_0}\right]$$

$$B_V=-\frac{\mu_0 I_p}{4\pi R_0}\left\{\frac{1}{\mu_0}\frac{\partial L_e}{\partial R}+\frac{\ell_i}{2}+\beta_p-\frac{1}{2}\right\}$$

$$M_V(\varepsilon,\kappa)=\frac{(1-\varepsilon)^2}{(1-\varepsilon)^2 c(\varepsilon)+d(\varepsilon)\sqrt{\kappa}}$$

$$c(\varepsilon)=1+0.98\varepsilon^2+0.49\varepsilon^4+1.47\varepsilon^6$$

$$d(\varepsilon)=0.25\varepsilon\left(1+0.84\varepsilon-1.44\varepsilon^2\right)$$

$$V_{LHI}=\frac{A_{inj}B_{\varphi,inj}}{\Psi}V_{inj}$$

$$V_{IR}=I_pR_p=I_p\left(\frac{\langle\eta_{spitzer}\rangle 2\pi R_0}{A_p}\right)$$

$$V_{geo}=-\frac{d}{dt}\left[L_eI_p\right]=-L_e\frac{dI_p}{dt}-I_p\frac{dL_e}{dt}$$

$$L_e=\mu_0R_0\frac{a(\varepsilon)(1-\varepsilon)}{1-\varepsilon+\kappa b(\varepsilon)}$$

$$a(\varepsilon)=\left(1+1.81\sqrt{\varepsilon}+2.05\varepsilon\right)\ln\left(\frac{8}{\varepsilon}\right)-\left(2.0+9.25\sqrt{\varepsilon}+1.21\varepsilon\right)$$

$$b(\varepsilon)=0.73\sqrt{\varepsilon}\left(1+2\varepsilon^4-6\varepsilon^5+3.7\varepsilon^6\right)$$

$$V_{W_m}\approx-\frac{1}{I_p}\frac{d}{dt}\left(\frac{1}{2}L_iI_p^2\right)$$

$$\ell_i=\frac{C_p^2}{\mu_0V_p}L_i$$

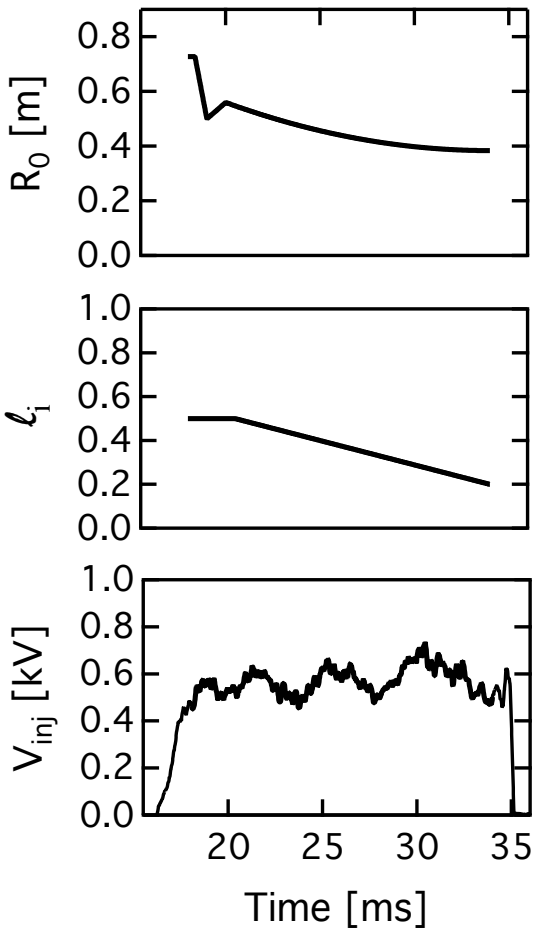
* S.P. Hirshman and G.H. Nielson 1986 Phys. Fluids **29** 790

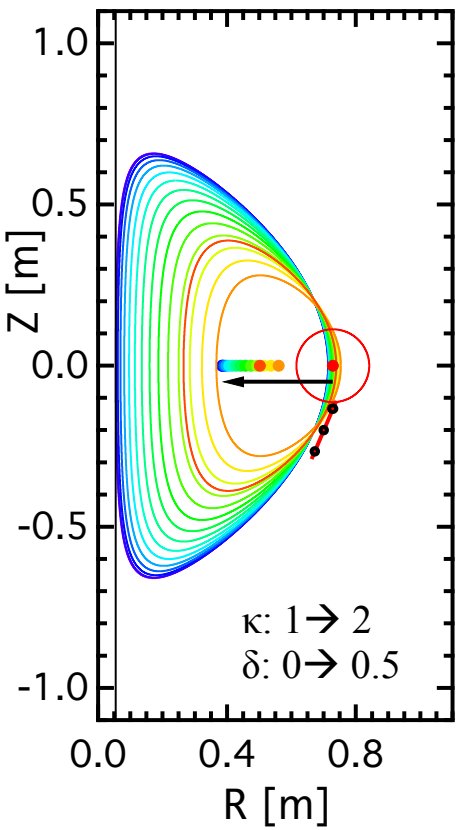
** O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*
S. Ejima et al 1982 Nucl. Fusion **22** 1313
J.A. Romero and JET-EFDA Contributors 2010 Nucl. Fusion **50** 115002

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0-D Model Takes Plasma, Injector Parameters as Inputs

- Initial condition: $I_p(t_0)=I_{TL}$
- Shape(t)
  - $R_0(t)$, $a(t)$, $\kappa(t)$, $\delta(t)$
  - Vertical symmetry
- $\langle\eta\rangle(t)$, $\ell_i(t)$, $\beta_p(t)$
  - Constant $\langle\eta\rangle$ assumed
    - Spitzer
  - $\beta_p = 0$
  - ℓ_i dropping: 0.5 → 0.2
- Injector Inputs:
  - $A_{inj}(t)$, $V_{inj}(t)$, $R_{inj}(t)$

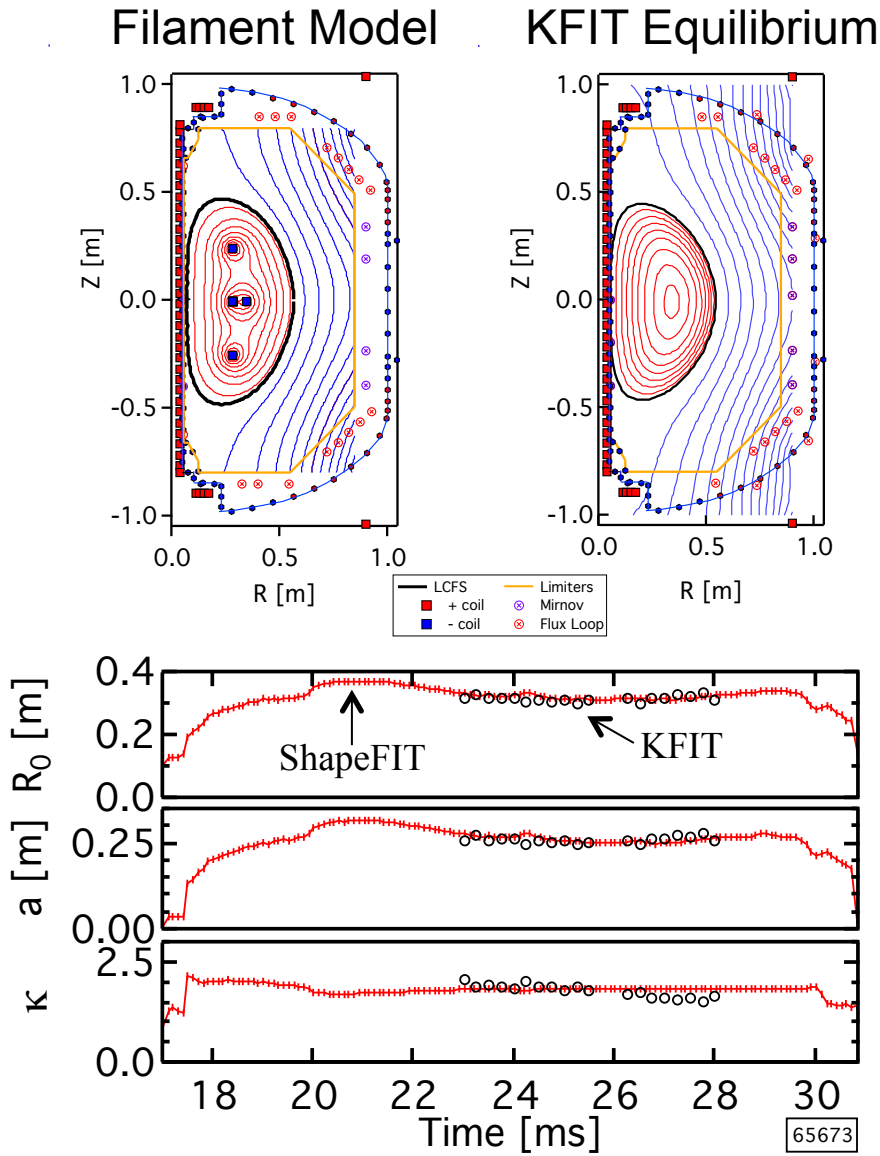




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ShapeFIT: Fast Boundary Reconstruction Code Provides Shape(t)

- Plasma treated as 4-6 filaments
  - Positions, currents fit to magnetics
- Wall currents modeled
  - Same model used in KFIT equilibrium code
- Validating against reconstructions
  - $R_0 \pm 1.5$ cm
  - $a \pm 1.5$ cm
  - $\kappa \pm 15\%$
  - $\delta \pm 25\%$

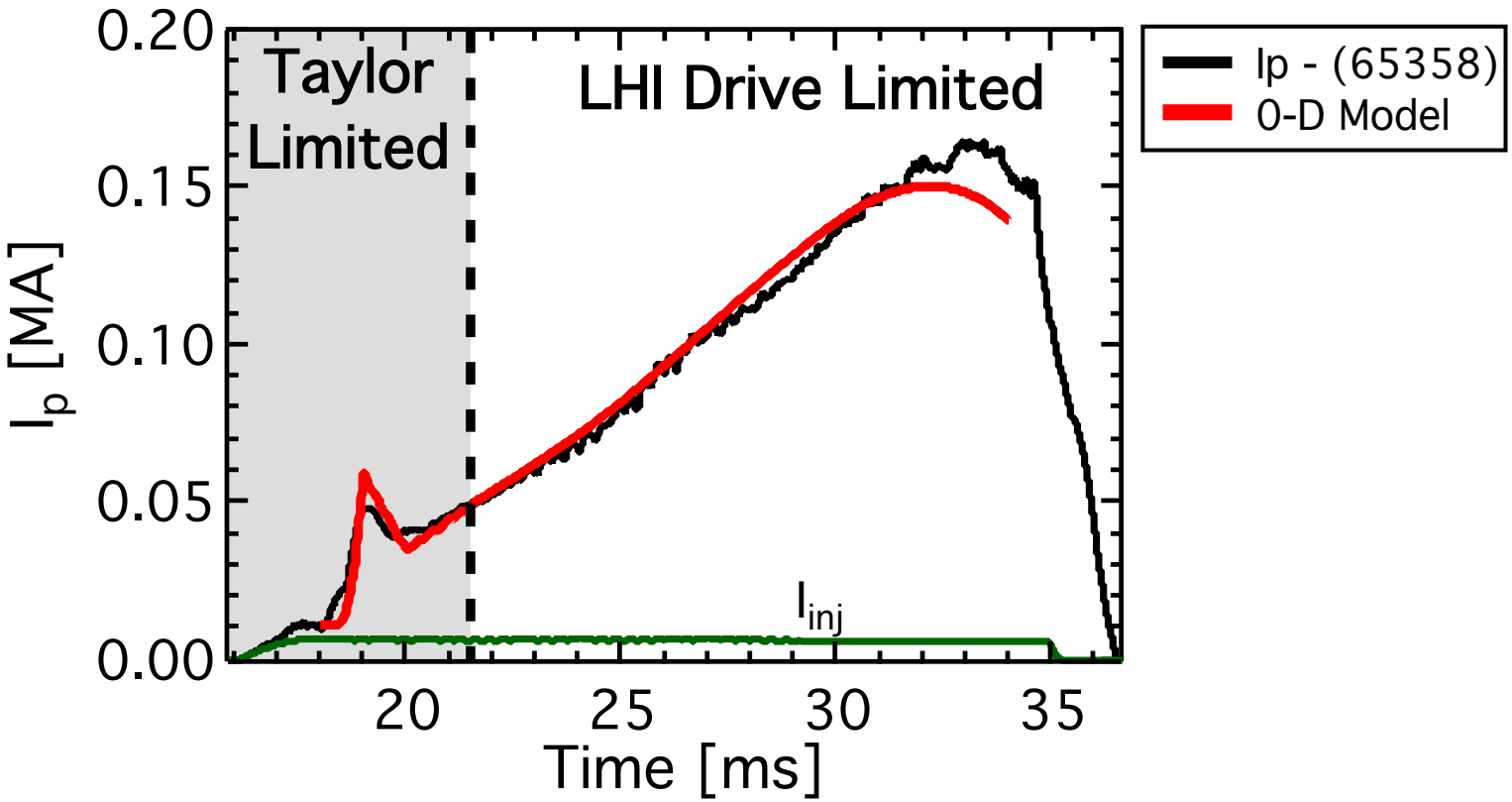


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Testing Model Against Experiment in PEGASUS

LHI Discharges Undergo Two-Phase $I_p(t)$ Evolution

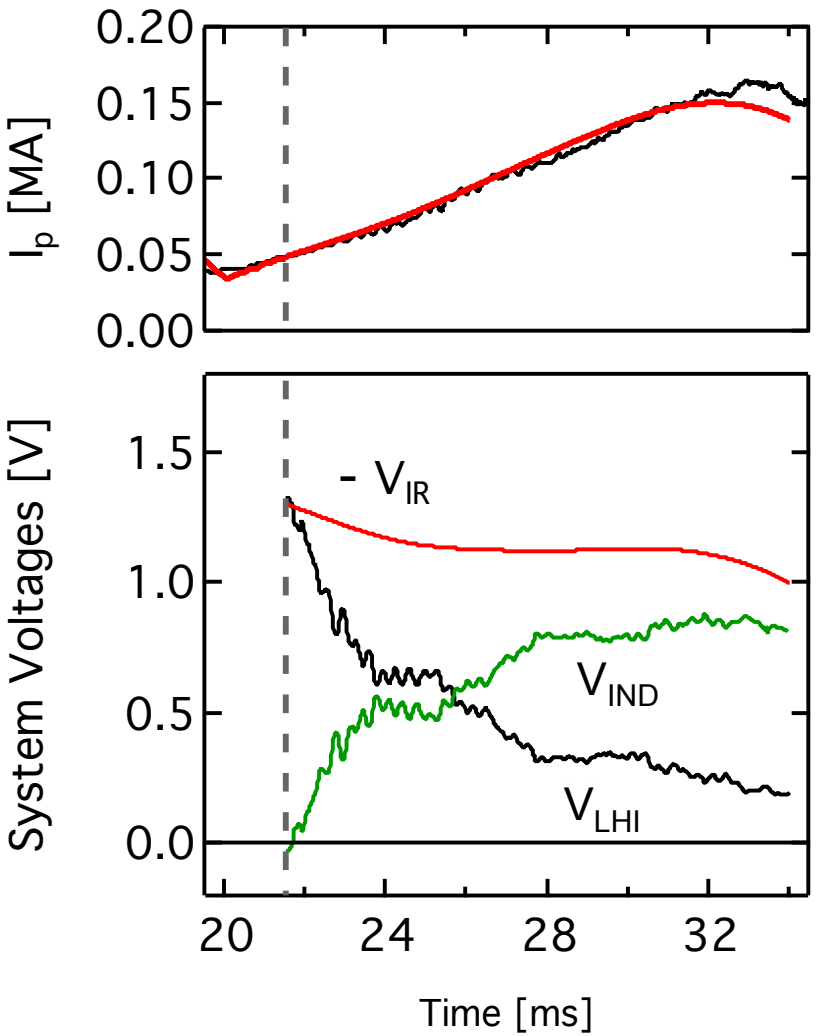


- Low I_p: subject to the Taylor Limit
- Higher I_p: power-balance, drive-limited

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Inward Expansion Provides Large Inductive Drive

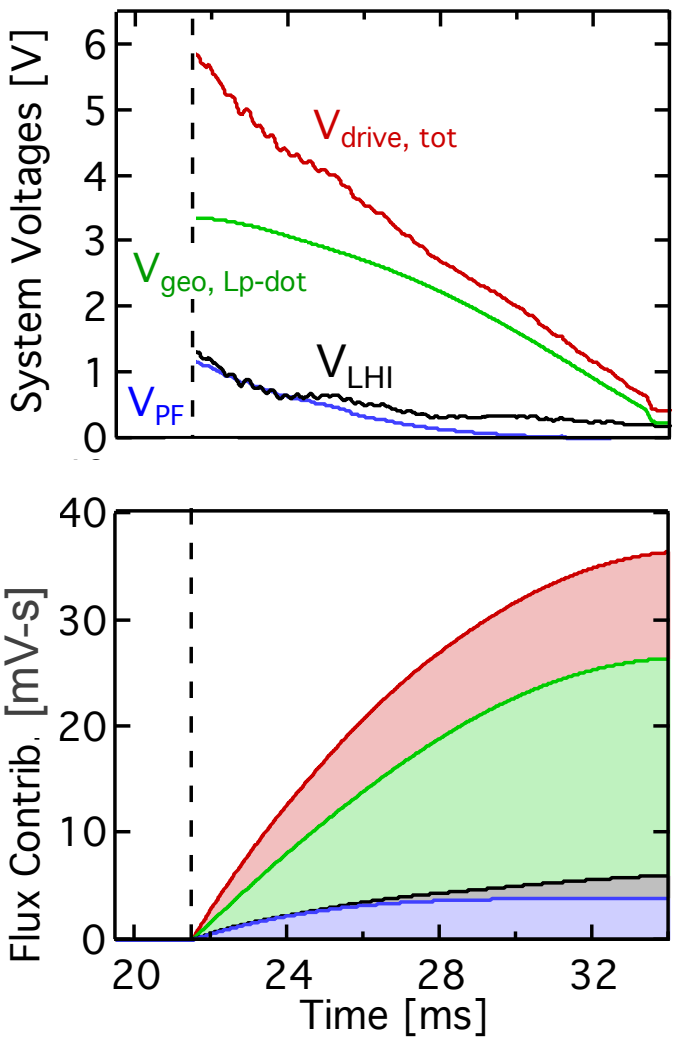
- Resistive losses primarily balanced by V_{IND}
  - V_{LHI} declines with plasma growth to large-size, low-A
- Inductive part includes both drive and anti-drive terms



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Geometric Induction Provides Majority Current Drive in Outboard LHI

Breakdown of drive voltages:



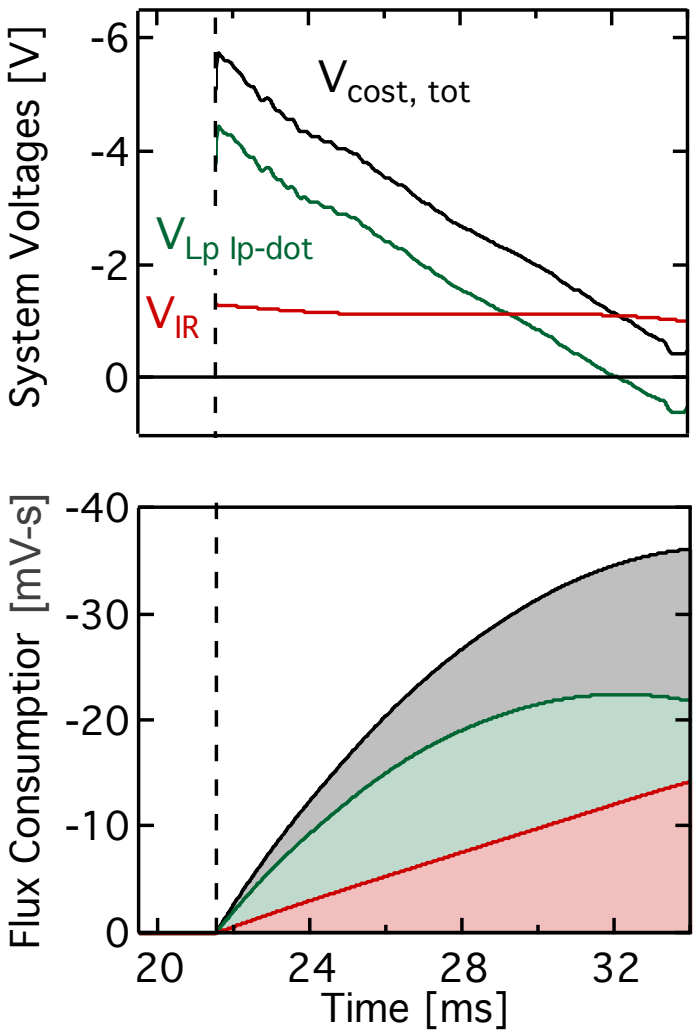
- **Geometry evolution**
  - Inductance drops with inward expansion
  - $V_{Ip\ Lp\text{-}dot}$: < 4V
  - ~73% of total drive V-s
- **V_{LHI}**
  - V_{LHI}: < 1 V
  - ~16% total V-s
- **PF ramping**
  - Necessary for force-balance
  - V_{PF}: ~1–2 V
  - ~11 % total V-s

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Drive Terms Balance Resistive Losses and Inductive Reactance

Breakdown of anti-drive voltages:



- **Resistive dissipation**
  - V_{IR}: ~1–2 V
  - ~40% of V-s
- **Inductive reactance**
  - $V_{Lp\ Ip\text{-}dot}$: ~ 4–6 V
  - ~60% of V-s
- Total: ~0.5–6V over discharge evolution

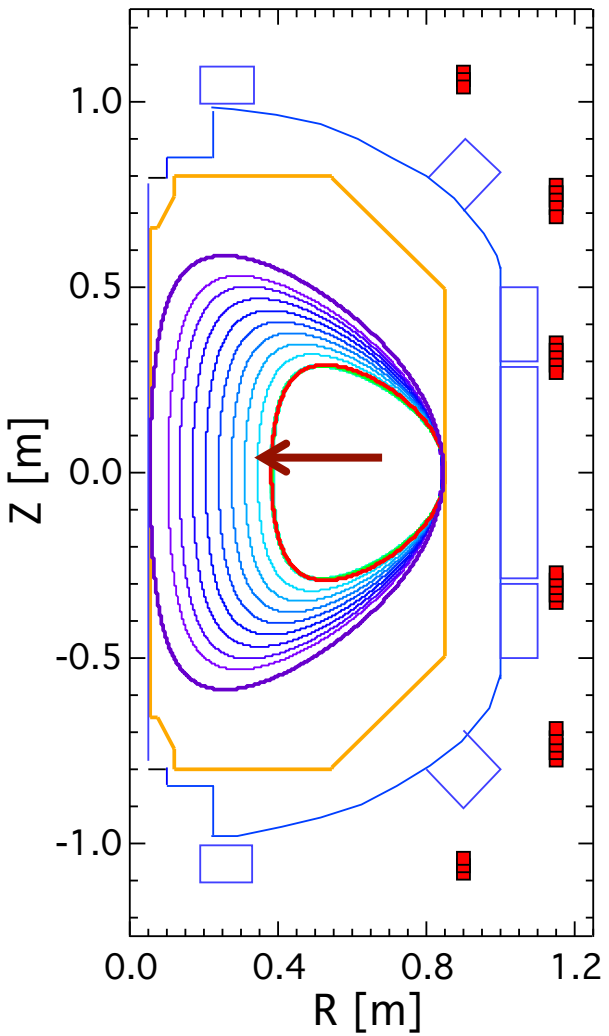
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Varying Shape(t) to Test Model

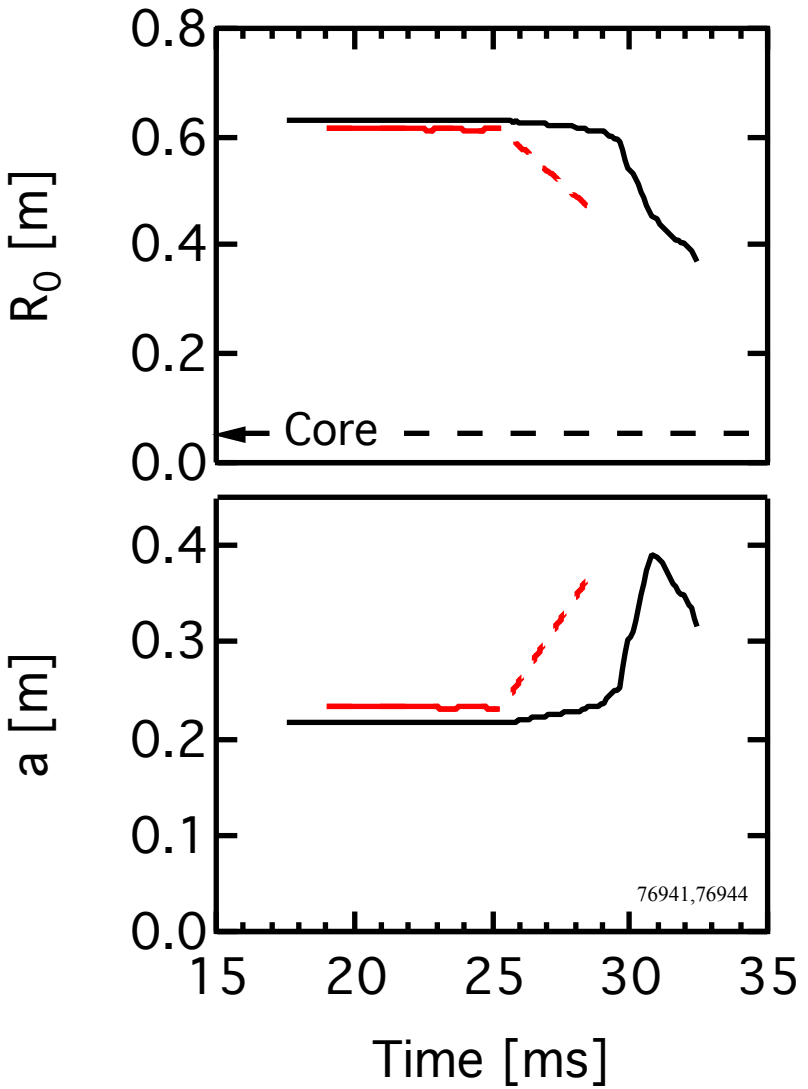
- Experiment conducted to widely vary V_{IND}
  - Stationary, outboard-limited plasmas
  - Controlled rapid expansion

- ShapeFIT used in control room to achieve desired shape evolutions



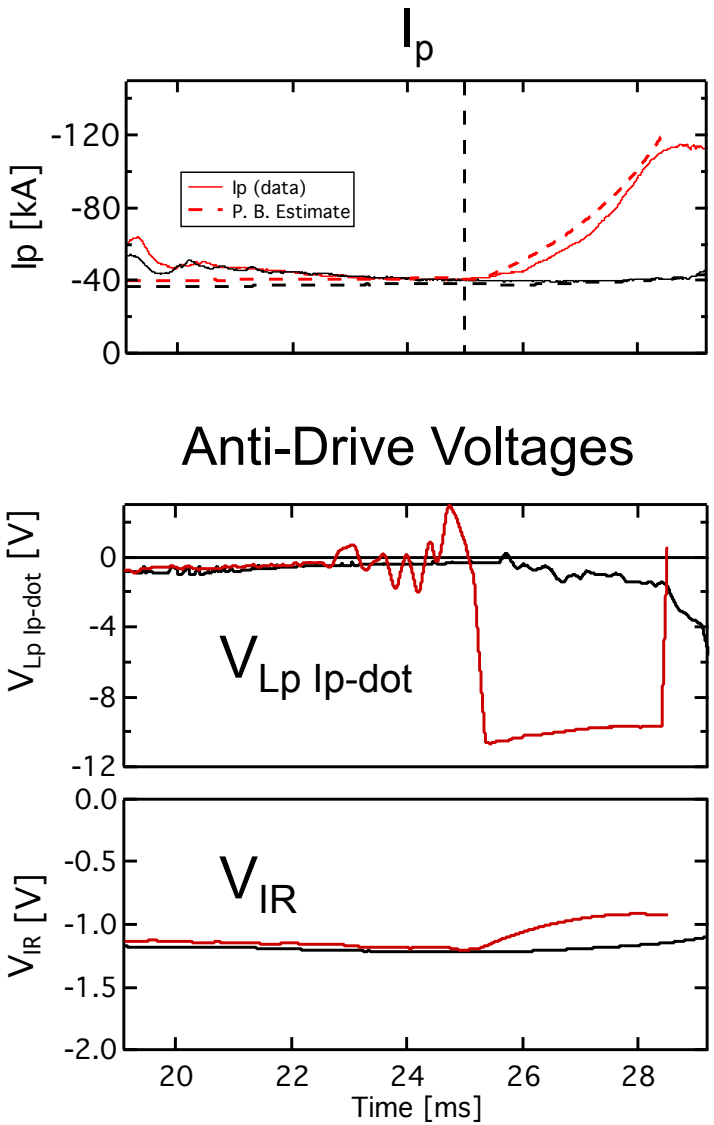
Desired Shape Evolutions Realized in Experiment

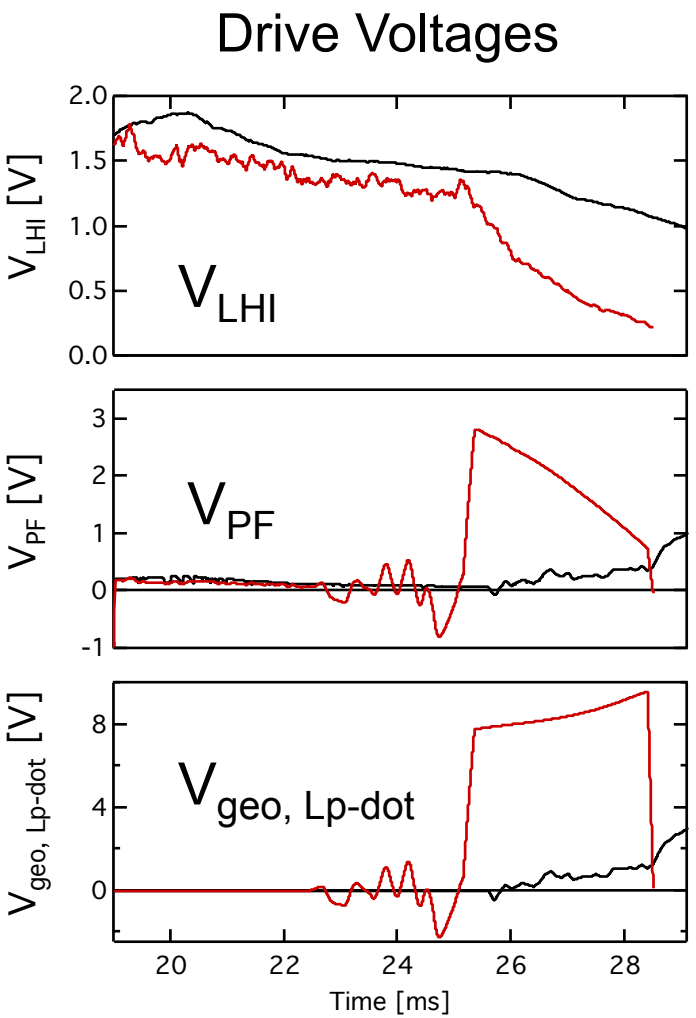
- Static, outboard-limited:
  - Minimal V_{IND}
  - Maximum V_{LHI}
- Rapid inward expansion:
  - Large V_{IND}
  - Dropping V_{LHI}



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Shape Control Greatly Varied Inductive Voltages

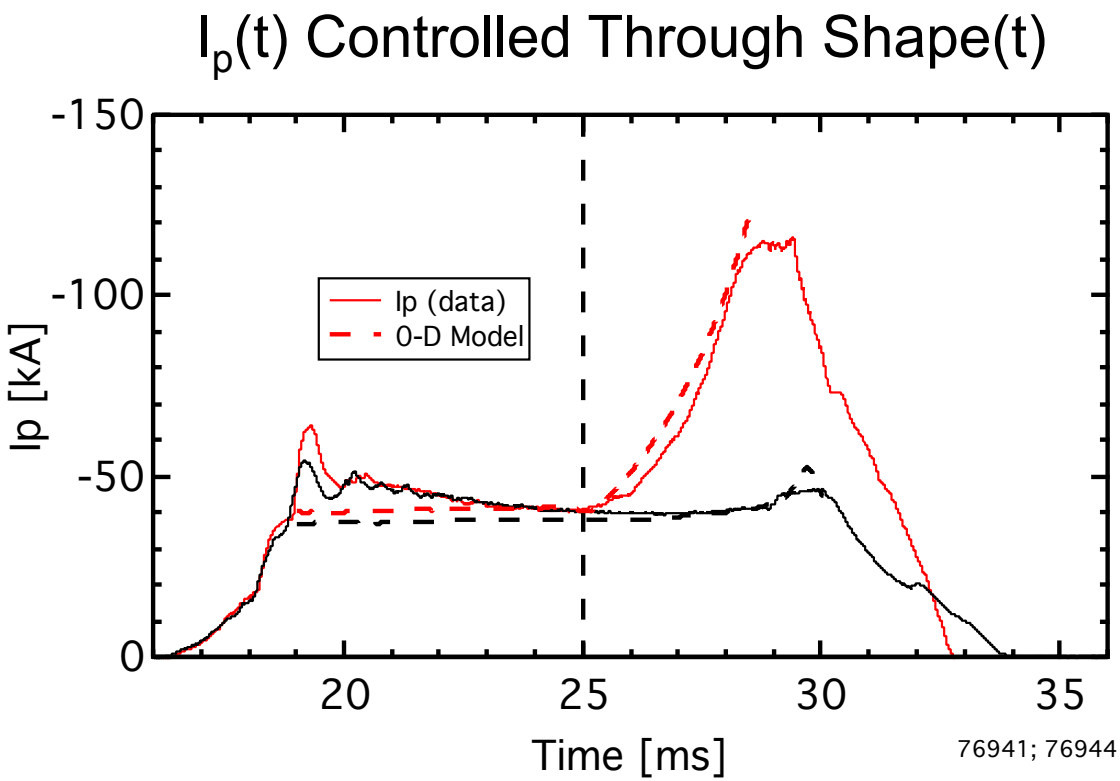




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0-D Model Reproduces $I_p(t)$ with Drastically Different Inductive Evolutions



- Inward expansion takes advantage of:
  - Drive from dropping L_p
  - Increased PF induction (force-balance)
  - Higher Taylor limit

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Path to Validation

Model Testing & Calibration Proceeding on Multiple Fronts

$$I_p V_s = \frac{\partial}{\partial t}\left(W_{B,p}\right) + I_p^2 R_p - I_p V_{NICD}$$

- $I_p(t)$ compared for various shape(t)

Plasma surface voltage

$$V_s = -\frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i} + L_e I_p\right)$$

- L_p formulae comparison to A~1 equilibria

Internal magnetic energy

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_{B,p}\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2} L_i I_p^2\right)$$

- Comparison to experimental reconstruction

Resistive Dissipation

$$V_R = I_p R_p = I_p\left(\frac{\eta_p 2\pi R_0}{A_p}\right)$$

- Quantify with Thomson Scattering

Non-inductive current drive

$$V_{NICD} = V_{LHI}$$

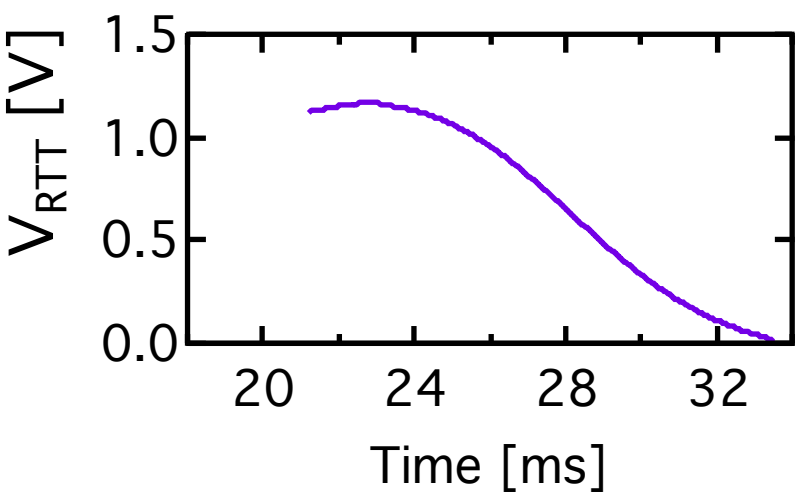
- Vary injector geometry for V_{LHI} dominance

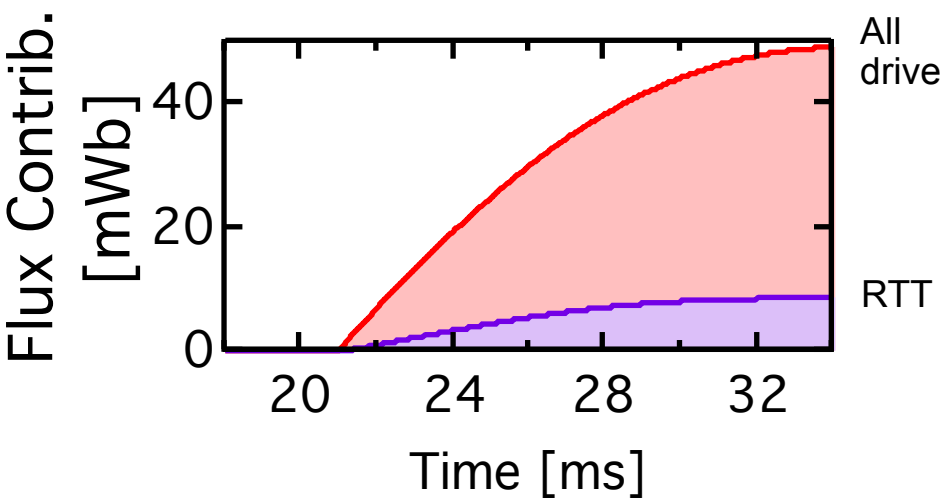
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Full Accounting of Moving Boundary is Required

- Reynolds' Transport Theorem applied to $\frac{\partial}{\partial t}W_m$ calculations

$$\iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right)dV = \frac{\partial}{\partial t}\left(\frac{1}{2}L_i I_p^2\right) - \oiint_{S_p} \frac{B_\theta^2}{2\mu_0}\vec{v}_b \bullet \hat{n}\, dA$$





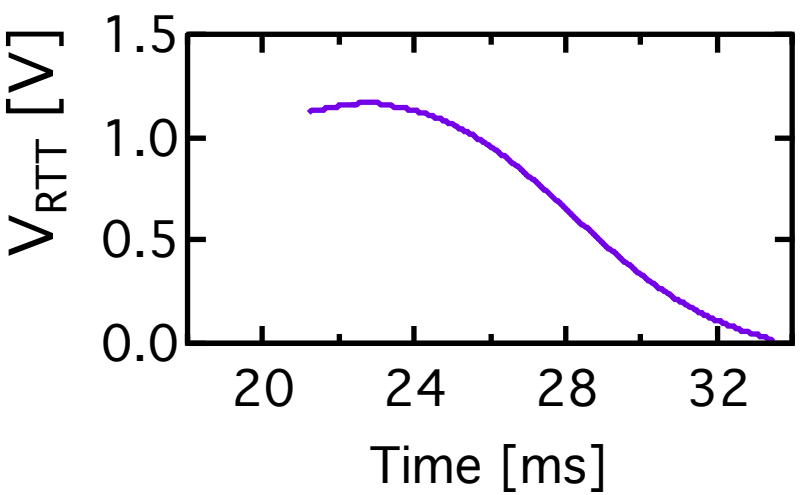
Suggests higher resistivity:
$\langle T_e \rangle$ ~ 60eV → ~ 50eV

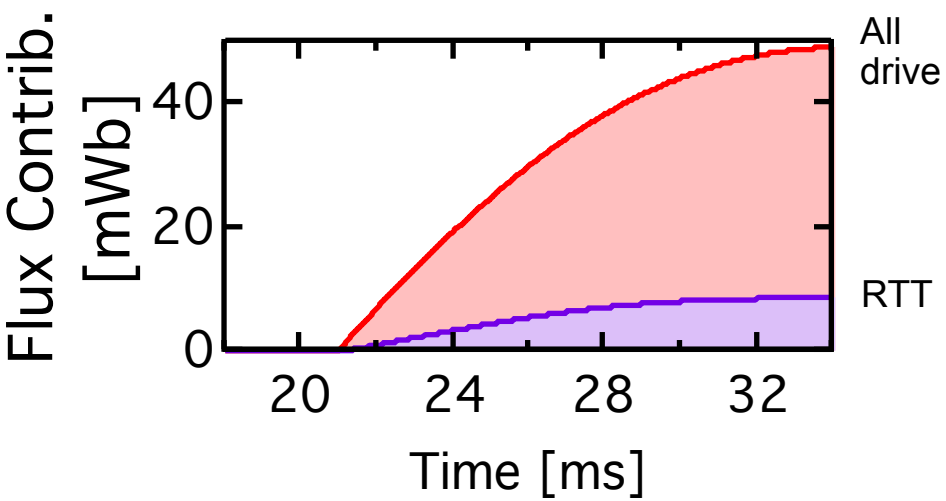
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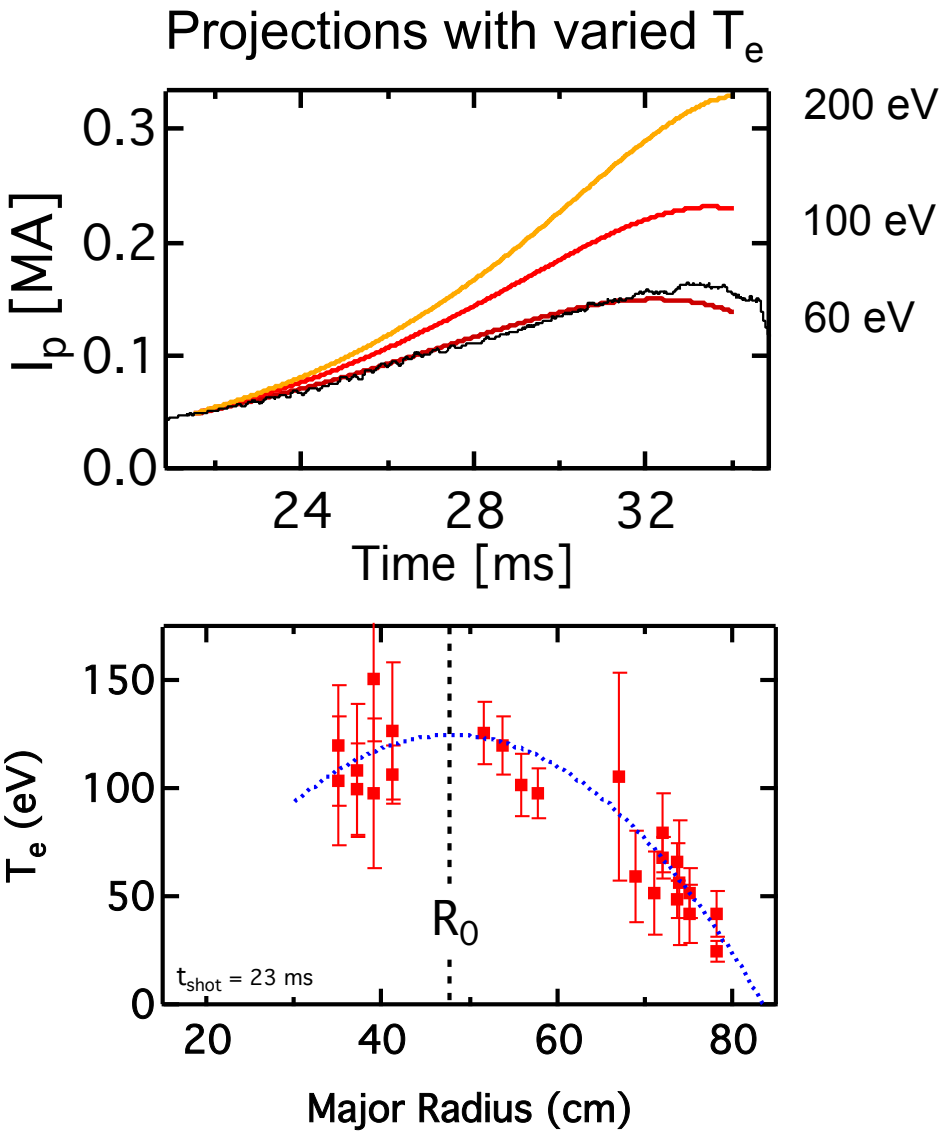




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Understanding Transport & Confinement is Needed for Scaling to Larger Experiments

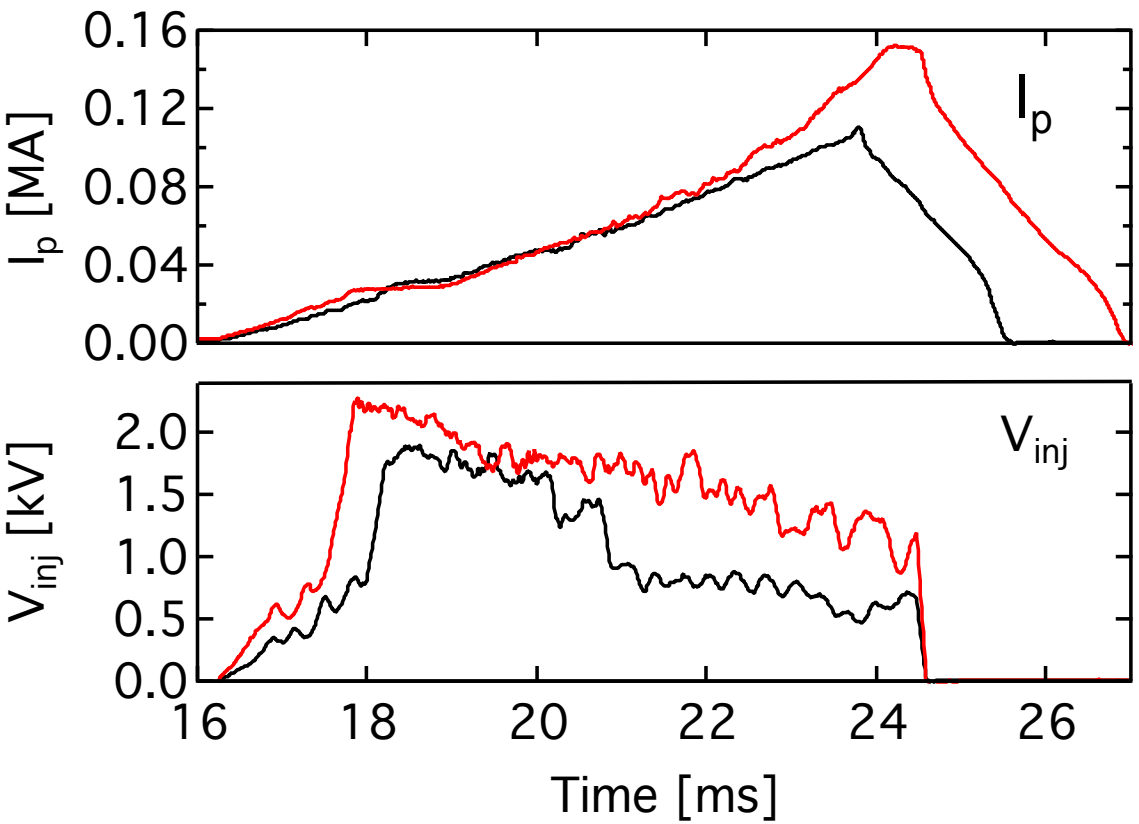
- Projections dependent strongly on assumed resistivity
- Transport, confinement determine resistive losses
  - Important for V_{LHI} requirements
- Initial Thomson Scattering profile indicates warm, tokamak-like core*
  - $\langle T_e \rangle_{volume}$ ~85 eV
  - Spitzer, uniform J(r,z): $\langle T_e \rangle$ ~75eV



* D.J. Schlossberg: #118
* G.M. Bodner: #119

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Gains in I_p Trend with Added V_{LHI}



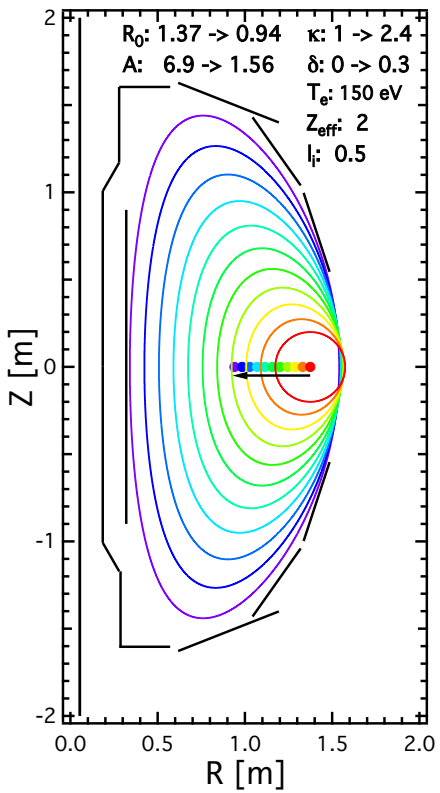
- Performance gains with experimental V_{LHI}

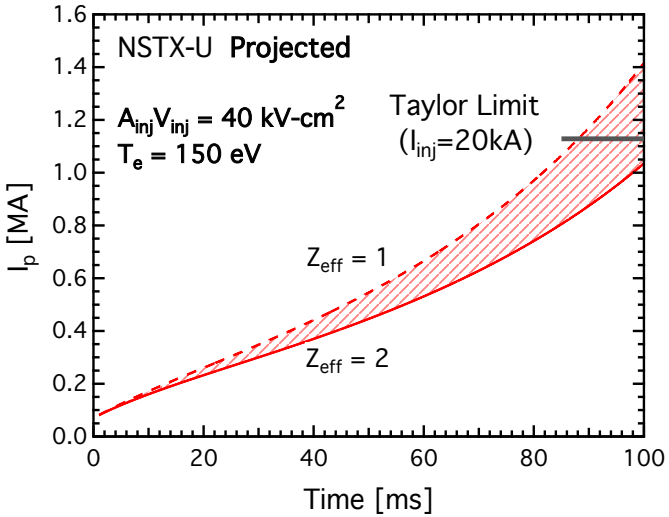
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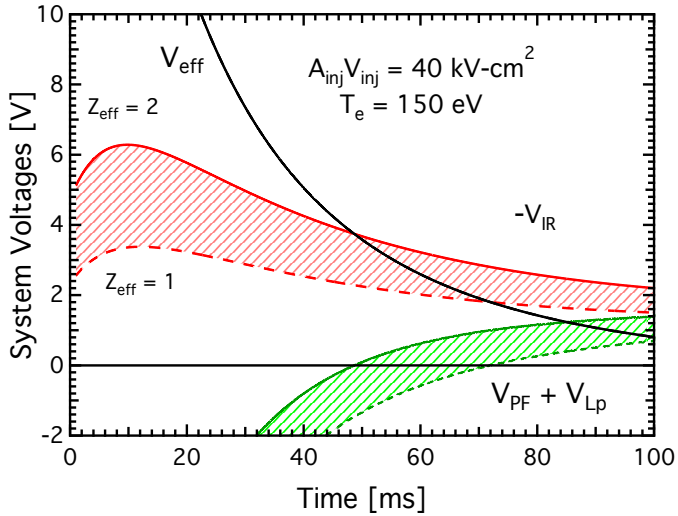


LHI Startup Scalability

Model Applied to NSTX-U Geometry for I_p~1 MA Scenario







- Significant LHI drive required to achieve I_p ~ 1 MA
  - Conservative confinement assumed
- Confinement scaling with LHI drive critical

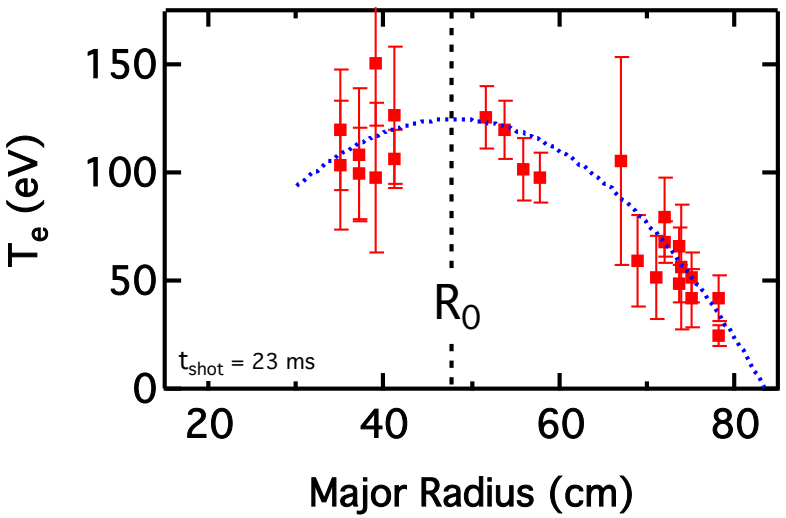
*C. Neumeyer *et all* (2009) *23rd IEEE/NPSS Symposium on Fusion Engineering*
**C. Neumeyer (2001) "NSTX Internal Hardware Dimensions"
http://nstx.pppl.gov/nstx/Engineering/NSTX_Eng_Site/Technical/Machine/NSTX_Engr_Machine_Dims_cm.html

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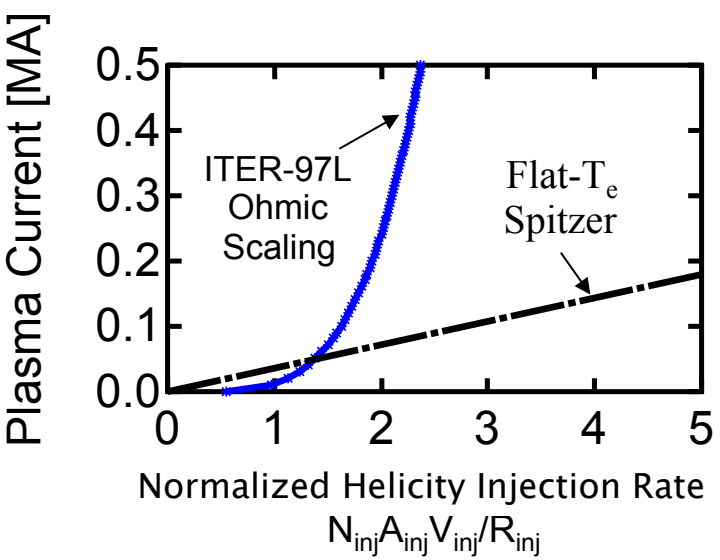
Strong V_{IND} Drive May Provide Good Core Confinement, Favorable Scaling

- First PEGASUS TS profile in LHI: $T_e(0)$ peaked in core
  - Profile is not highly stochastic
  - $T_e(0)$ comparable to Ohmic L-mode
- May indicate two zone confinement
  - Edge vs. core
- Divertor-injectors being deployed to isolate confinement effects
  - Static geometry
  - High V_{LHI}

*J.M. Perry: PO6.00001
*J.A. Reusch: GP12.00113



Effect of Confinement Scaling



A Predictive Power-Balance Model for LHI Startup is Being Validated on PEGASUS

- Model reasonably recreates experimental $I_p(t)$ with varied shape evolutions
- Outboard LHI uses large inductive drive from inward expansion
  - Provides > 70% of V-s in current experiment
- Understanding confinement, transport during LHI is important for projection to larger devices
  - Future experiments with divertor injectors
- I_p~1 MA LHI startup on NSTX-U appears feasible with conservative confinement assumptions
  - Motives further confinement study with high helicity injection rates