A Predictive Model for Non-solenoidal Startup using Local Helicity Injection

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PEGASUS Toroidal Experiment

Outline

- Local Helicity Injection (LHI) provides routine $I_p \leq 0.18$ MA non-solenoidal startup on Pegasus
  - LHI discharges readily handoff to OH H-mode sustainment
- Outboard LHI startup discharges undergo dynamic geometric, inductive evolutions
- A predictive, lumped-parameter, power-balance model is under development to account for these dynamics
  - Model is being experimentally validated, calibrated on Pegasus
- Model predicts $I_p \sim 1$ MA NSTX-U startup will be in a regime where LHI drive dominates current ramp phase
  - Same regime attainable on Pegasus at $I_p \sim 0.3$ MA with injector, facility upgrades

*Work supported by US DOE Grants DE-FG02-96ER54375 and DE-SC0006928

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Helicity Injection Provides Tokamak Current Drive

Total helicity K in a Tokamak geometry: $K = \int_V \left(\mathbf{A} + \mathbf{A}_{vac}\right) \cdot \left(\mathbf{B} - \mathbf{B}_{vac}\right) \mathrm{d}^3\mathrm{x}$

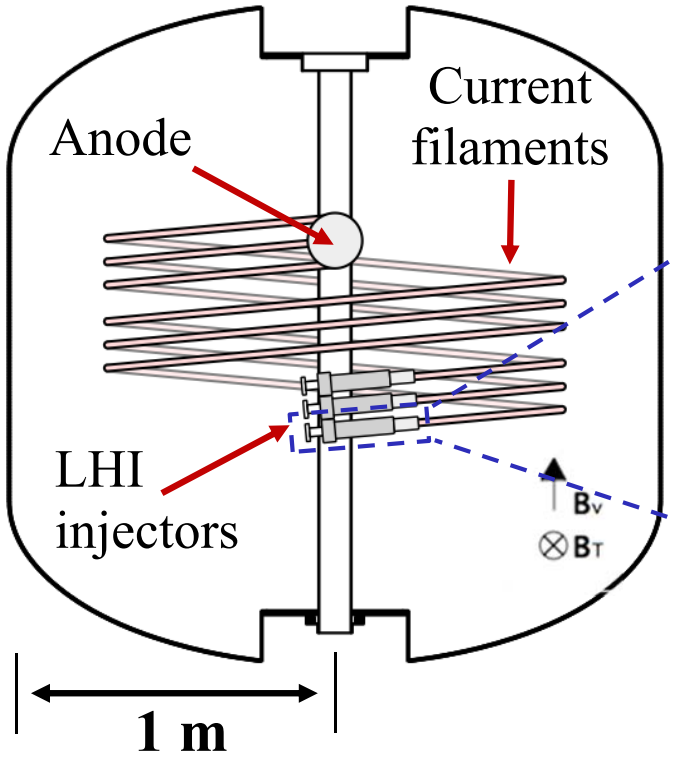
$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J} \cdot \mathbf{B}\, \mathrm{d}^3\mathrm{x} - 2\frac{\partial \psi}{\partial t}\Psi - 2\int_A \Phi \mathbf{B} \cdot \mathrm{d}\mathbf{s}$$

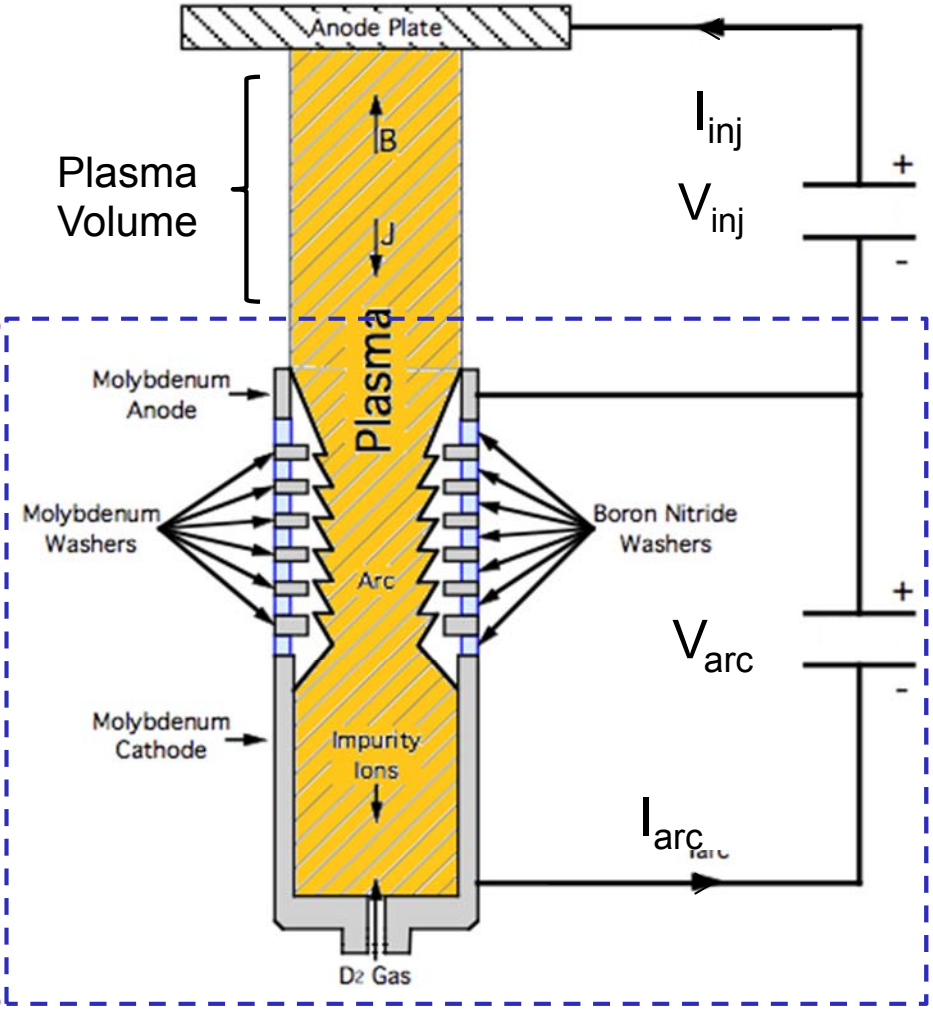
- Resistive Helicity Dissipation
  - $\mathbf{E} = \eta\mathbf{J} \rightarrow$ much slower than energy dissipation ($\eta\mathbf{J}^2$)
  - Turbulent relaxation processes dissipate energy and conserve helicity
- AC Helicity Injection: $\dot{K}_{AC} = -2\frac{\partial \psi}{\partial t}\Psi = 2V_{loop}\Psi$
  - Ψ is toroidal flux, ψ is poloidal flux (*e.g.*, current drive through solenoid induction)
- DC Helicity Injection $\dot{K}_{DC} = -2\int_A \Phi \mathbf{B} \cdot \mathrm{d}\mathbf{s} = 2V_{bias}B_{\perp}A_{inj}$
  - Φ is electrostatic potential

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Helicity Injection with Edge Localized Current Injectors

- Outboard LHI systems inject current, helicity on low-field-side
- Scalable “port-plug” design
- Assisted by PF ramping for additional current drive





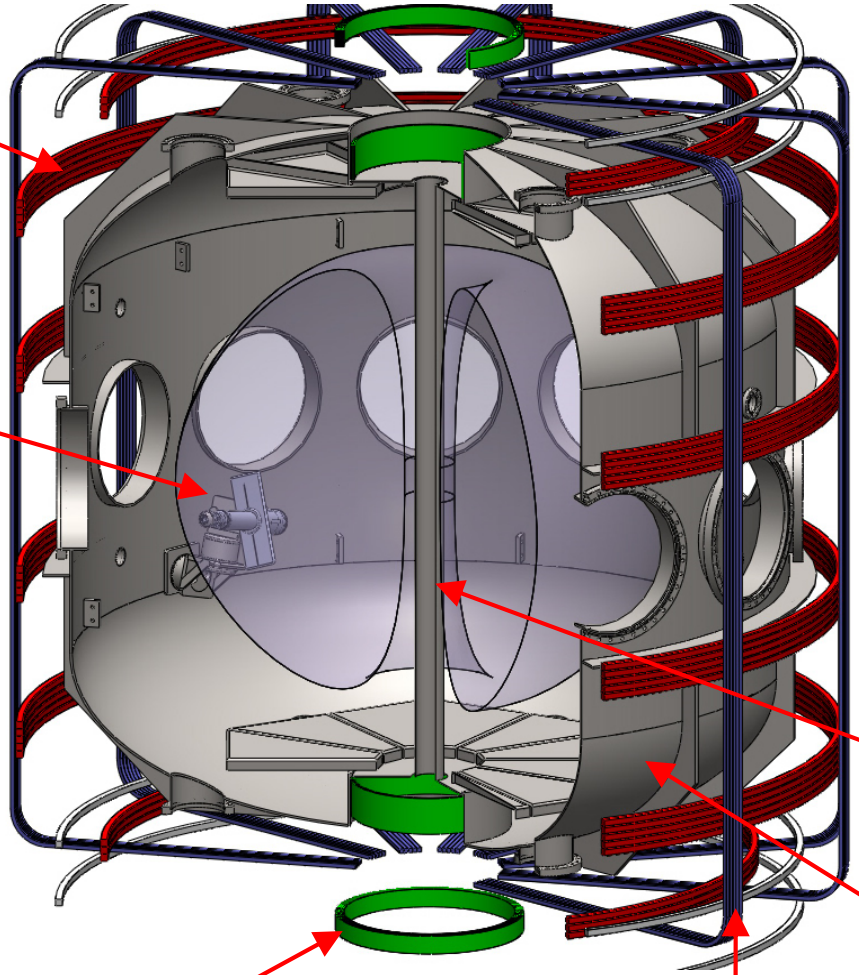
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Outboard LHI Provides Robust $I_p \leq 0.18$ MA Startup on the PEGASUS ST

Equilibrium Field Coils

Local DC Helicity Injectors





High-stress Ohmic heating solenoid

Vacuum Vessel

Divertor Coils

Toroidal Field Coils

Plasma Parameters

I_p	≤ 0.18 MA
τ_{shot}	≤ 0.025 s
B_T	0.15 T
A	1.15 – 1.3
R	0.2 – 0.45 m
a	≤ 0.4 m
κ	1.4 – 3.7

Injector Parameters

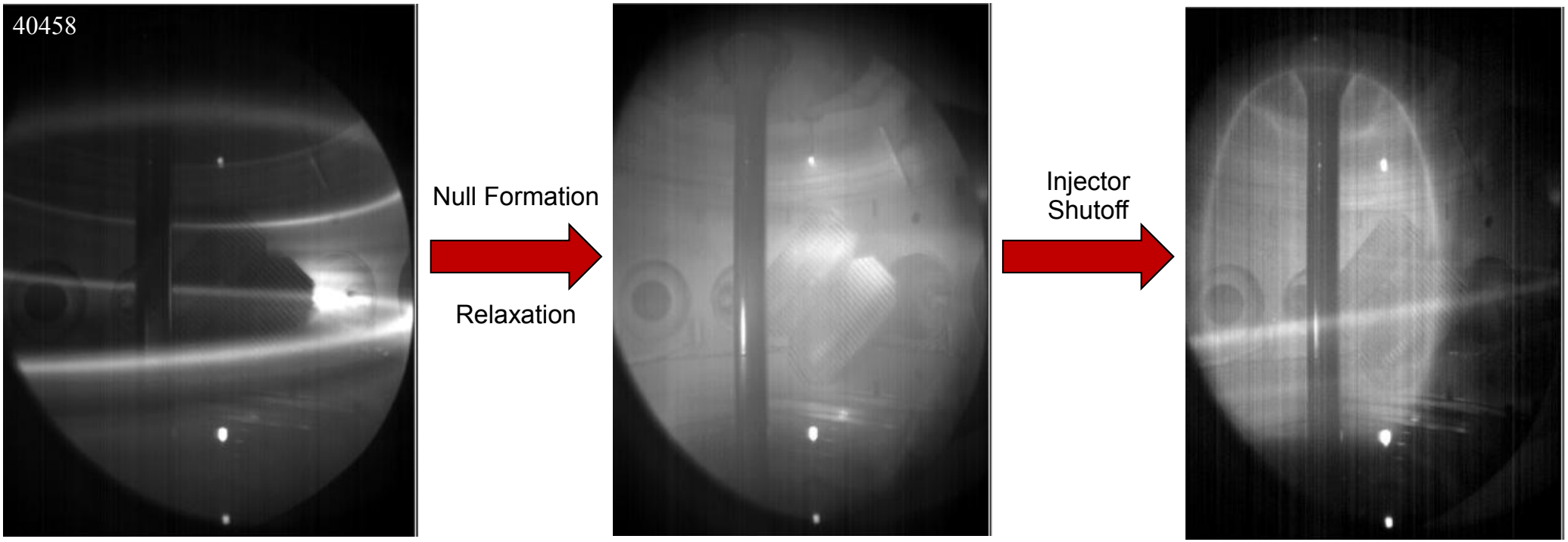
I_{inj}	≤ 6.5 kA
V_{inj}	≤ 2.5 kV

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LHI Startup is Dynamic

LHI Injects Current Streams that Relax, Form Tokamak-Like Plasma



- Current injected along helical vacuum field
  - Local, active current sources
- MHD relaxation, tokamak-like state
  - Onset via local PF null
  - Constrained by helicity, Taylor relaxation limits
- Tokamak plasmas produced after injector shut off
  - Couples to alternative current drive sources

*Battaglia et al., Nucl. Fusion **51**, 073029 (2011)*

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Helicity Balance, Taylor Relaxation Criteria Set Max Achievable I_p from LHI

Helicity balance in a Tokamak geometry:

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J}\cdot\mathbf{B}\, d^3x - 2\frac{\partial\psi}{\partial t}\Psi - 2\int_A \Phi\mathbf{B}\cdot d\mathbf{s} \quad \Rightarrow \quad I_p \le \frac{A_p}{2\pi R_0\langle\eta\rangle}\left(V_{ind} + V_{eff}\right)$$

- Helicity injection provides an effective loop voltage:

$$V_{eff} \approx \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{inj}$$

Taylor relaxation of a force-free equilibrium:

$$\nabla\times B = \mu_0 J = \lambda B$$

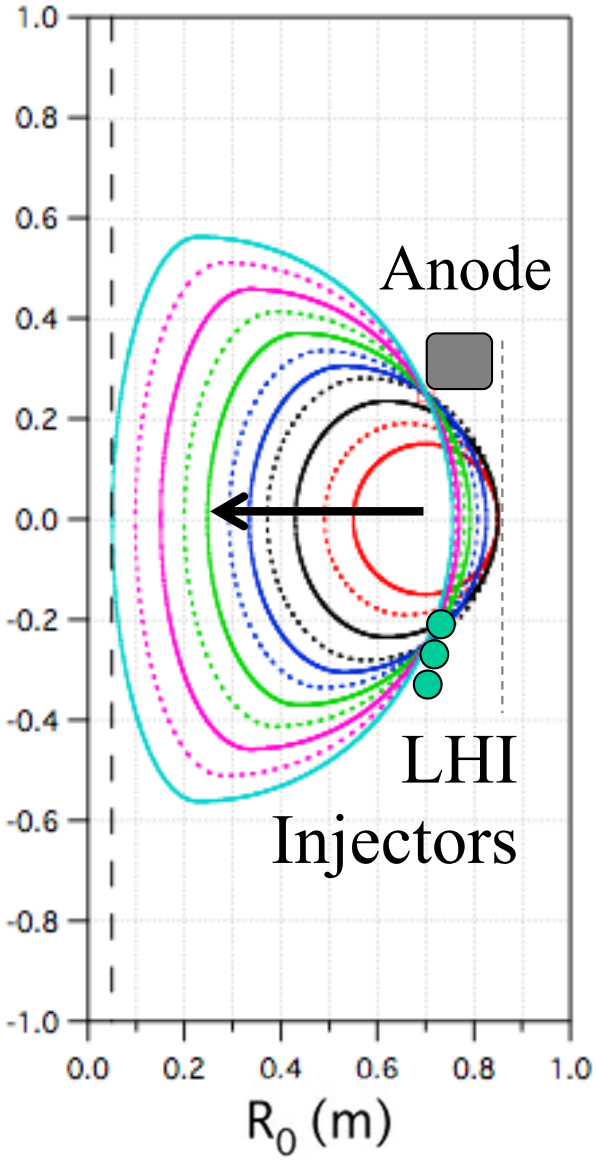
$$\lambda_p \le \lambda_{edge} \quad \Rightarrow \quad \frac{\mu_0 I_p}{\Psi_T} \le \frac{\mu_0 I_{inj}}{2\pi R_{inj} w B_{\theta+V,inj}} \quad \Rightarrow \quad I_p \le \sqrt{\frac{1}{B_{\theta+V,inj}/I_p}\frac{\Psi_T I_{inj}}{2\pi R_{inj} w_{inj}}}$$

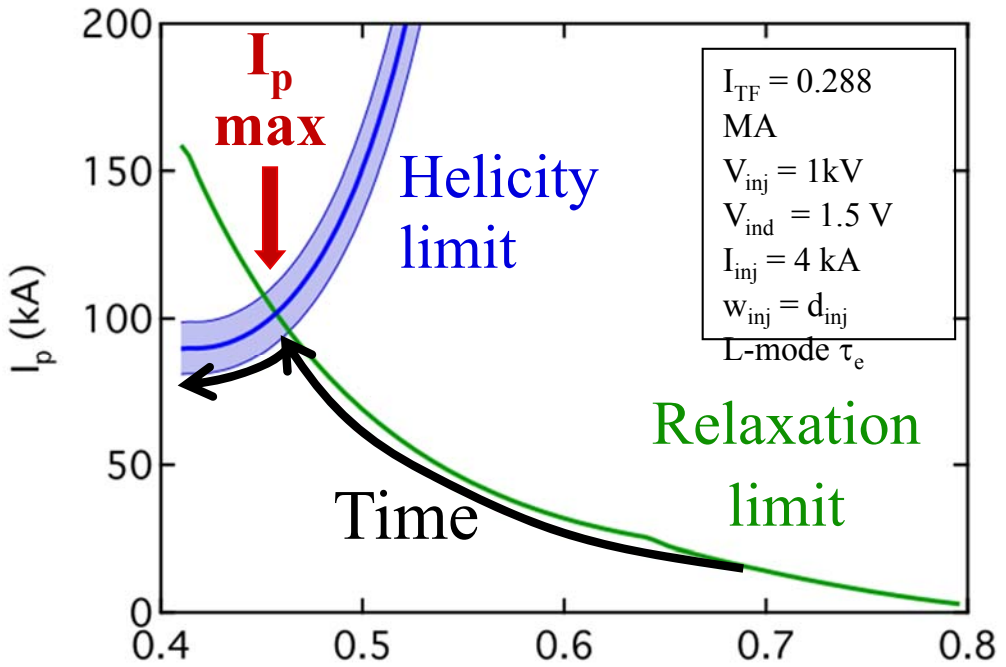
A_p	Plasma area	Ψ_T	Plasma toroidal flux
w_{inj}	Edge current channel width	$B_{\theta+V,inj}$	Poloidal field in injection region

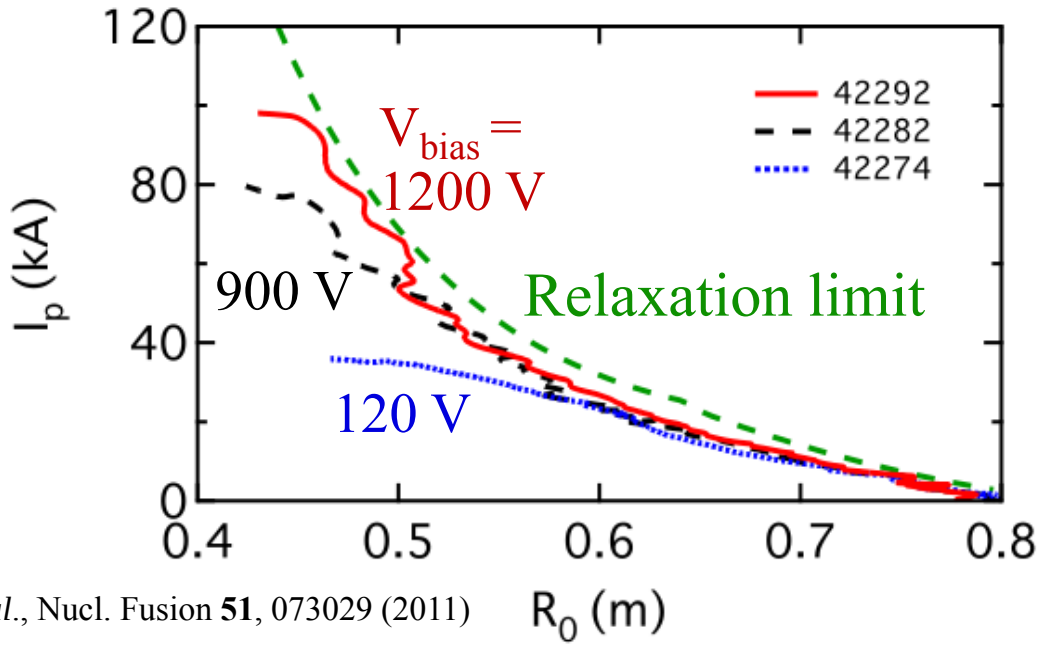
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Sufficient Helicity Injection is Required to Drive Plasma to the Taylor Limit

Estimated plasma evolution:







*Battaglia *et al.*, Nucl. Fusion **51**, 073029 (2011)

Predicting LHI $I_p(t)$ Requires Accounting of Dynamic Plasma Geometry Evolution

- Estimating injector requirements for scalable, high performance, LHI startup requires a predictive $I_p(t)$ model

- LHI plasmas execute dynamic geometric, inductive evolution
  - The Taylor relaxation current limit gives the maximum attainable I_p, but does not predict V_{eff} to balance resistive and inductive system voltages

- A power-balance model is under development to guide injector and scenario development
  - Multi-faceted effort testing, calibrating to Pegasus experimental data

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A Power-Balance Model for LHI Startup

Taylor Relaxation Theory, Power-Balance Applied to Predict LHI $I_p(t)$ Dynamics

0-D LHI model predicts $I_p(t)$ based on lowest of two limits:

- Taylor relaxation provides an absolute I_p limit:
  - Calculated from injector parameters, plasma/injector geometry*

$$I_p \leq \sqrt{\frac{1}{B_{\theta+V,inj}/I_p}\frac{\Psi_T I_{inj}}{2\pi R_{inj} w_{inj}}}$$

*Battaglia *et al.*, Nucl. Fusion **51**, 073029 (2011)

- Simple $I_p(t)$ circuit model derived from Poynting's Theorem:
  - Applied at the plasma boundary
  - Under development, testing at Pegasus

$$I_p V_s \approx \iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right) dV + I_p^2 R_p - I_p V_{NICD}$$

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Poynting's Theorem Models Power-Balance

$$I_p V_s = \iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right) dV + I_p^2 R_p - I_p V_{NICD}$$

Plasma surface voltage

- Force-balance explicitly enforced
- Inductive current-drive from OH, PF-ramping
- Plasma external inductance

$$V_s = -\frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i} + L_e I_p\right)$$

Internal magnetic energy

- Plasma internal inductance
- Estimated with constant boundary for now, later application of Reynold's Transport Theorem

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_{B,p}\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2} L_i I_p^2\right)$$

Resistive Dissipation

- Spitzer resistivity assumed

$$V_R = I_p R_p = I_p\left(\frac{\langle \eta_p \rangle 2\pi R_0}{A_p}\right)$$

Non-inductive current drive

- From Local Helicity Injection

$$V_{NICD} = V_{eff} \approx \frac{A_{inj} B_{\phi,inj}}{\Psi_T} V_{bias}$$

*J.A. Romero and JET-EFDA Contributors 2010 Nucl. Fusion **50** 115002

*S.P. Hirshman and G.H. Nielson 1986 Phys. Fluids **29** 790

*S. Ejima et al 1982 Nucl. Fusion **22** 1313

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Plasma Inductance Includes Analytic, Low-*A* External Inductance Approximation

- Plasma self-inductance can be described as a combination of internal, external inductances:

$$L_p = L_e + L_i$$

- Plasma external inductance heavily aspect-ratio dependent

$$L_e = \mu_0 R_0 \frac{a(\varepsilon)(1-\varepsilon)}{1-\varepsilon+\kappa b(\varepsilon)}$$

$$a(\varepsilon) = \left(1+1.81\sqrt{\varepsilon}+2.05\varepsilon\right)\ln\left(\frac{8}{\varepsilon}\right)-\left(2.0+9.25\sqrt{\varepsilon}+1.21\varepsilon\right)$$

$$b(\varepsilon) = 0.73\sqrt{\varepsilon}\left(1+2\varepsilon^4-6\varepsilon^5+3.7\varepsilon^6\right)$$

*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790

- Internal inductance typically assumed decreasing during the discharge (typically ℓ_i = 0.5→0.2)

$$W_{B,p} = \iiint_{V_p} \frac{B_\theta^2}{2\mu_0} dV = \frac{1}{2} L_i I_p^2$$

$$\ell_i = \frac{C_p^2}{\mu_0 V_p} L_i \xrightarrow[\text{Circular}]{\text{High-A}} \frac{2L_i}{\mu_0 R_0}$$

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Low-*A*, Analytic Models Approximate V_{PF}

- Applied vertical field provides force-balance and inductive loop-voltage
  - Added ψ_s estimate with Hirshman & Nielson* mutual inductance-like M_v equation
  - Assumes uniform vertical-field

$$V_{PF} = \frac{\partial}{\partial t}\left(\sum_i \psi_{PF,i}\right) \approx \frac{\partial}{\partial t}\left[M_V \pi R_0^2 \, B_V\big|_{R_0}\right]$$

where $$M_V(\varepsilon,\kappa) = \frac{(1-\varepsilon)^2}{(1-\varepsilon)^2 c(\varepsilon) + d(\varepsilon)\sqrt{\kappa}}$$

$$c(\varepsilon) = 1 + 0.98\varepsilon^2 + 0.49\varepsilon^4 + 1.47\varepsilon^6$$

$$d(\varepsilon) = 0.25\varepsilon\left(1 + 0.84\varepsilon - 1.44\varepsilon^2\right)$$

*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790

- B_v required for force-balance is aspect-ratio and shape dependent
  - Discharges evolve from high-*A* to low-*A*, and circular to strongly shaped
  - Uses Mitarai & Takase* formula for B_v for force-balance at low-*A* with **κ**:

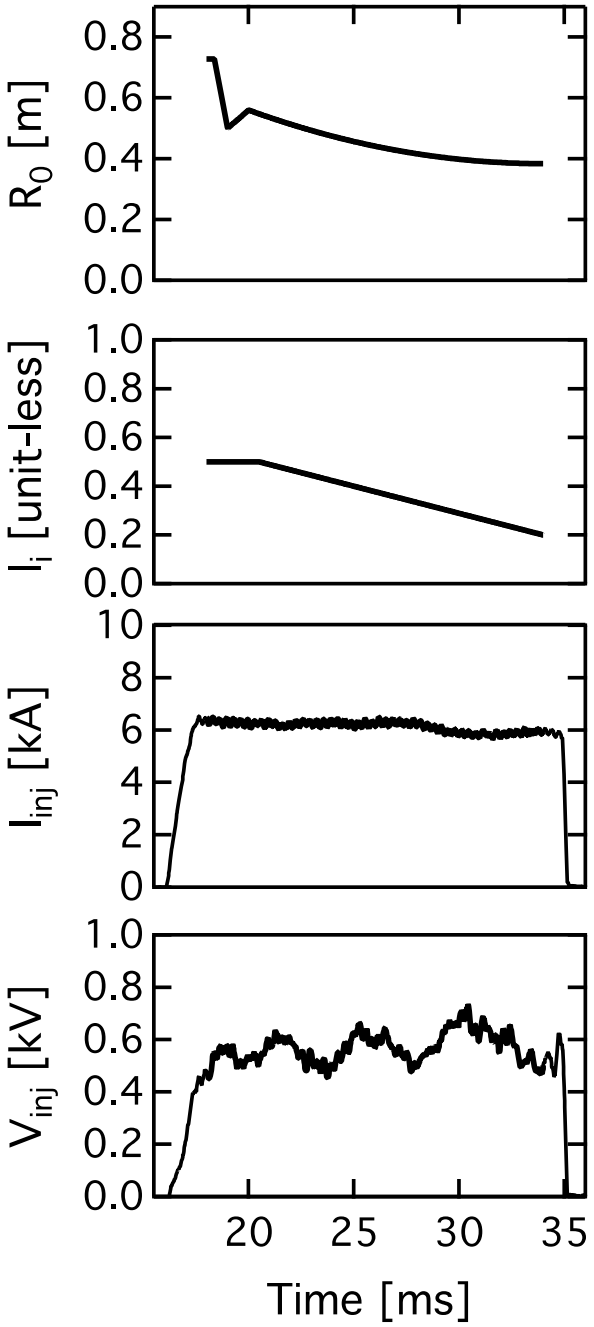
$$B_V = \frac{\mu_0 I_p}{4\pi R_0}\left\{\frac{1}{\mu_0}\frac{\partial L_e}{\partial R} + \frac{\ell_i}{2} + \beta_p - \frac{1}{2}\right\}$$

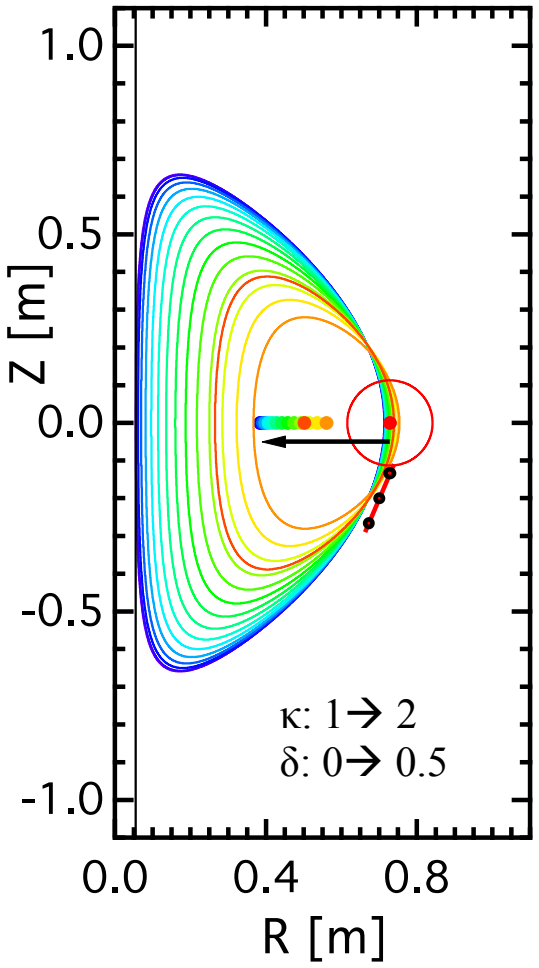
*O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*

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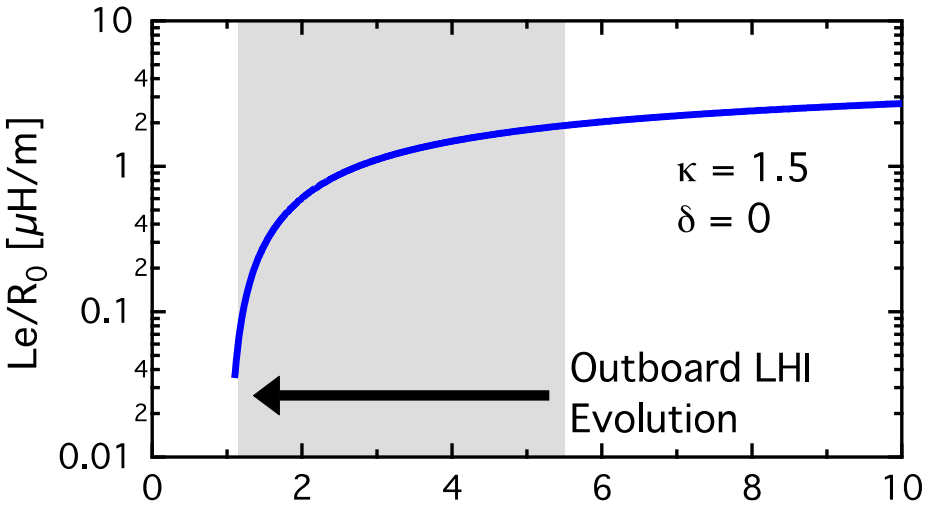
Plasma Parameters Constitute Lumped-Parameter Inputs

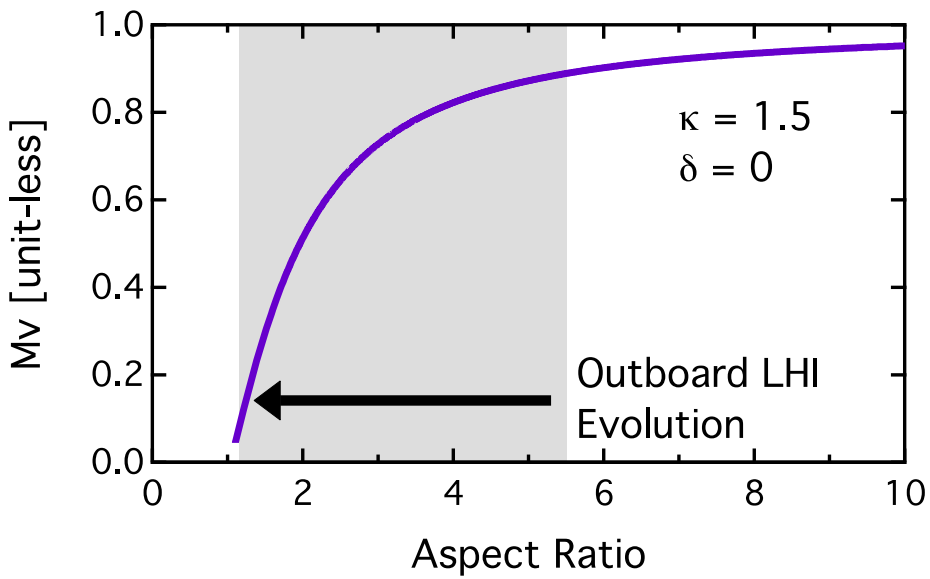
- $I_p(t_0)$
  - Initial condition to DE solver
- Shape
  - $R_0(t)$, $a(t)$, $\kappa(t)$, $\delta(t)$
  - Vertical symmetry ($Z_0(t)=0$)
  - Equilibrium reconstructions in progress
- $\langle\eta\rangle(t)$, $\ell_i(t)$, $\beta_p(t)$
  - $\langle\eta\rangle$ assumed Spitzer, constant in time, space for now
  - $\beta_p = 0$
  - ℓ_i dropping: 0.5 → 0.2
- Experimental $A_{inj}(t)$, $V_{inj}(t)$





L_e, M_v, are Highly Nonlinear in Aspect Ratio when $A \leq 2$





- Dropping L_e provides driving voltage:

$$V_{Le} \approx L_e \frac{\partial}{\partial t}\left[I_p\right] + I_p \frac{\partial}{\partial t}\left[L_e\right]$$

- Dropping M_v exerts anti-drive:
  - Diminishes PF ramping effectiveness

$$V_{PF} \approx \left(M_V \pi R_0^2 \frac{B_V|_{R_0}}{I_p} \right) \frac{\partial}{\partial t}\left[I_p\right] + I_p \frac{\partial}{\partial t}\left[M_V \pi R_0^2 \frac{B_V|_{R_0}}{I_p} \right]$$

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Testing and Calibration of the Model Proceeding on Multiple Fronts

$$I_p V_s = \frac{\partial}{\partial t}\left(W_{B,p}\right) + I_p^2 R_p - I_p V_{NICD}$$

- Predicted, experimental $I_p(t)$ compared for various plasma evolutions

Plasma surface voltage →

$$V_s = -\frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i} + L_e I_p\right)$$

Internal magnetic energy →

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_{B,p}\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2} L_i I_p^2\right)$$

- Approximations for $\psi_s(t)$, $W_p(t)$, $L_i(t)$ calibrated to experiment
  - Using time-evolving magnetic equilibrium reconstructions
- Multifilament magnetics analyzer for (early) plasma shape evolution

Resistive Dissipation →

$$V_R = I_p R_p = I_p\left(\frac{\eta_p 2\pi R_0}{A_p}\right)$$

- New Thomson Scattering diagnostic deployed to measure LHI confinement

Non-inductive current drive →

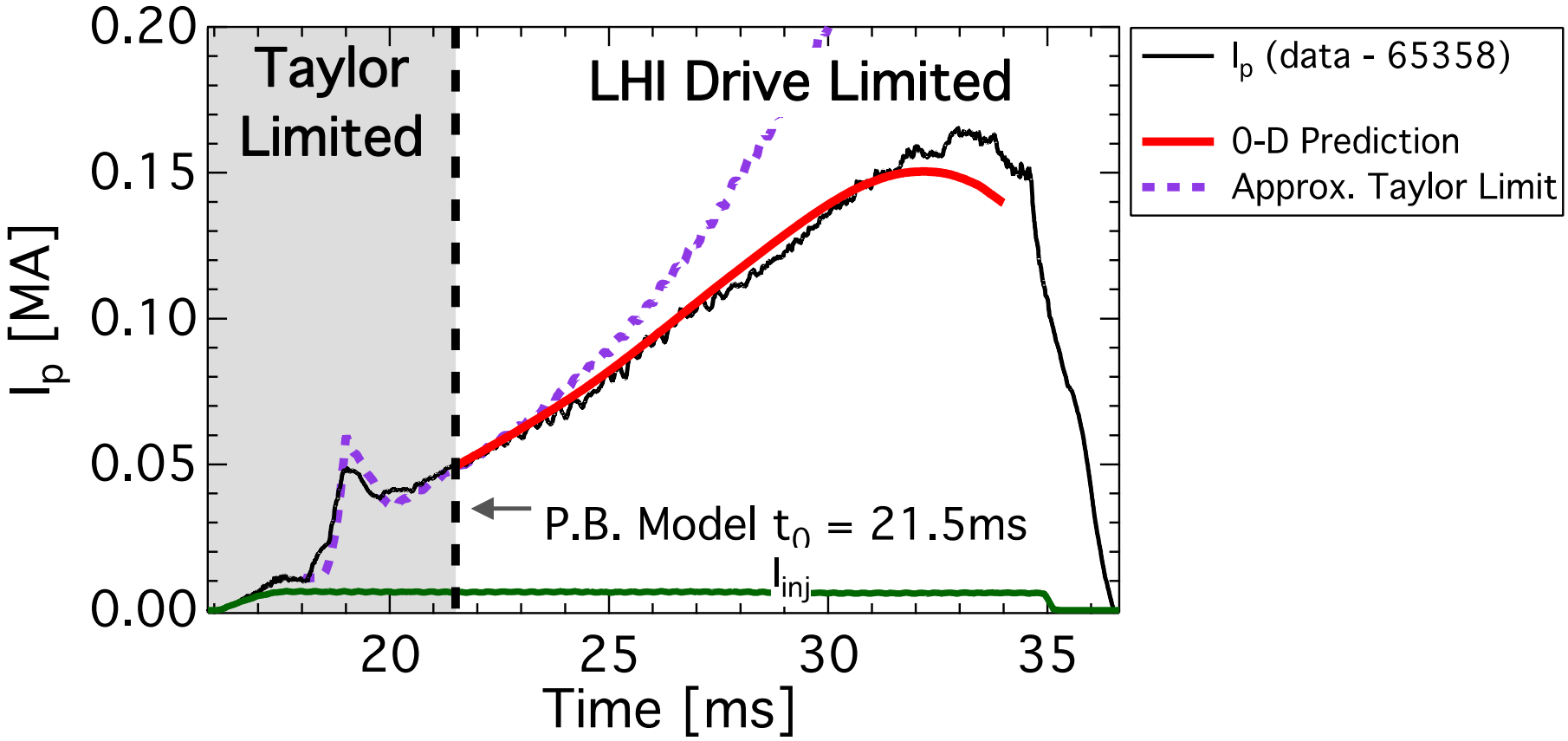
$$V_{NICD} = V_{eff}$$

- Upgraded injector systems to significantly vary V_{eff}

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0-D Model Reasonably Recreates Pegasus $I_p(t)$

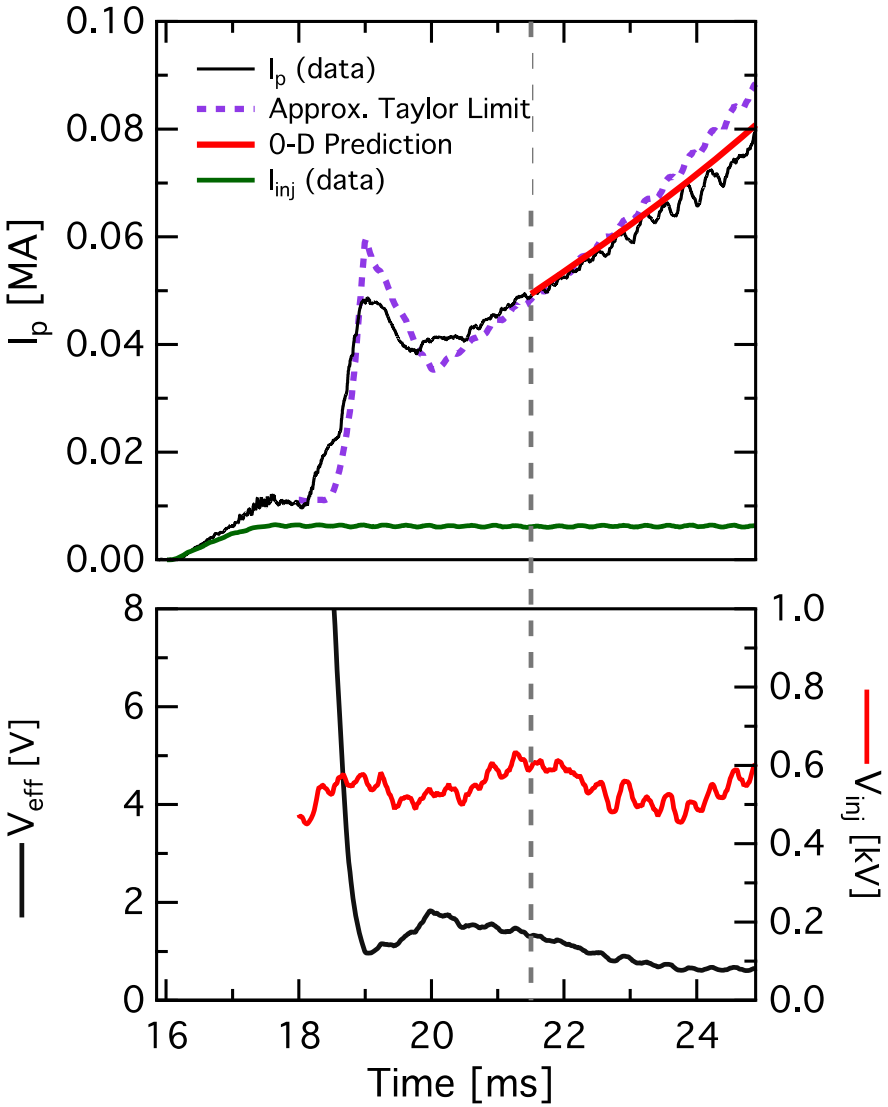
LHI Discharges Undergo a Two-Phase $I_p(t)$ Evolution



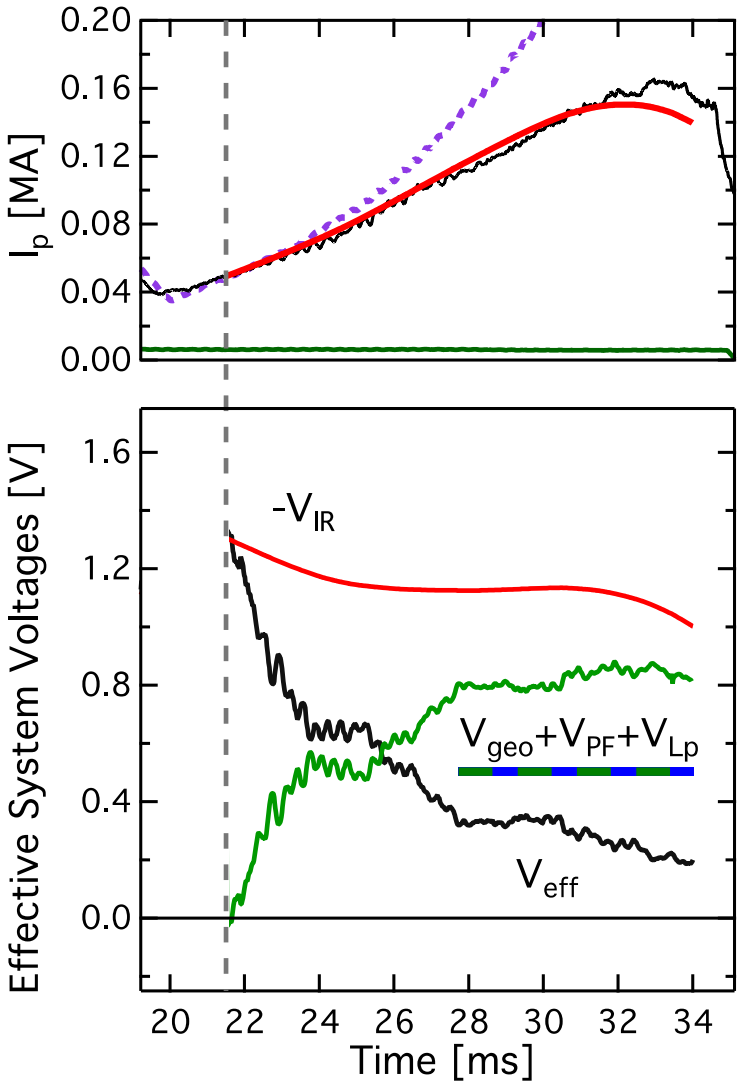
- Undergoes two-phase $I_p(t)$ evolution:
  - Low I_p: subject to the Taylor Limit
  - Higher I_p: power-balance, helicity limit

Early Phase Exploits Significant V_{eff}, Late $I_p(t)$ Relies on Inductive Effects

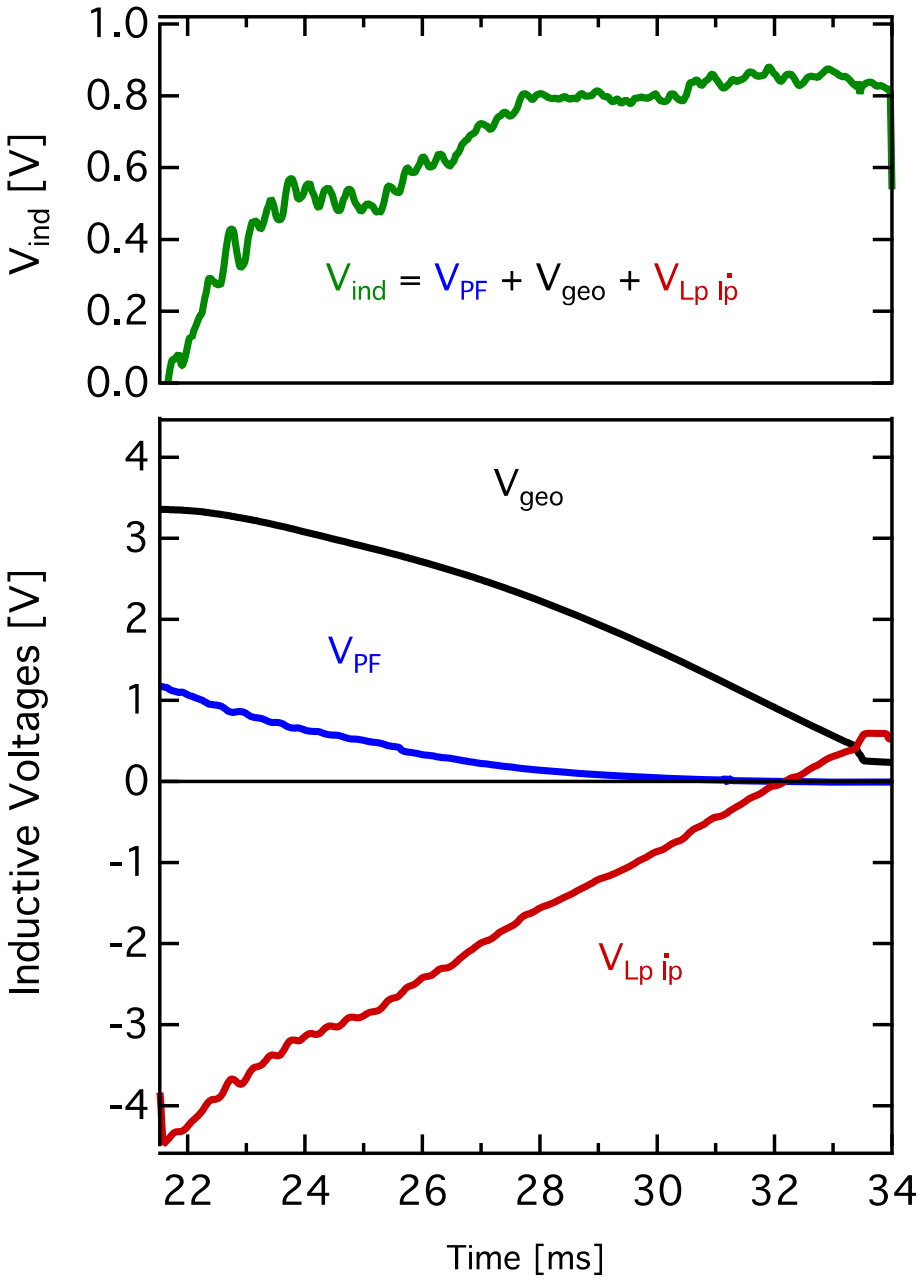
Sufficient LHI drive in early phase to access the Taylor Limit



Inductive, geometric effects dominate over LHI drive in later phase



Three Effects Contribute to Inductive Drive



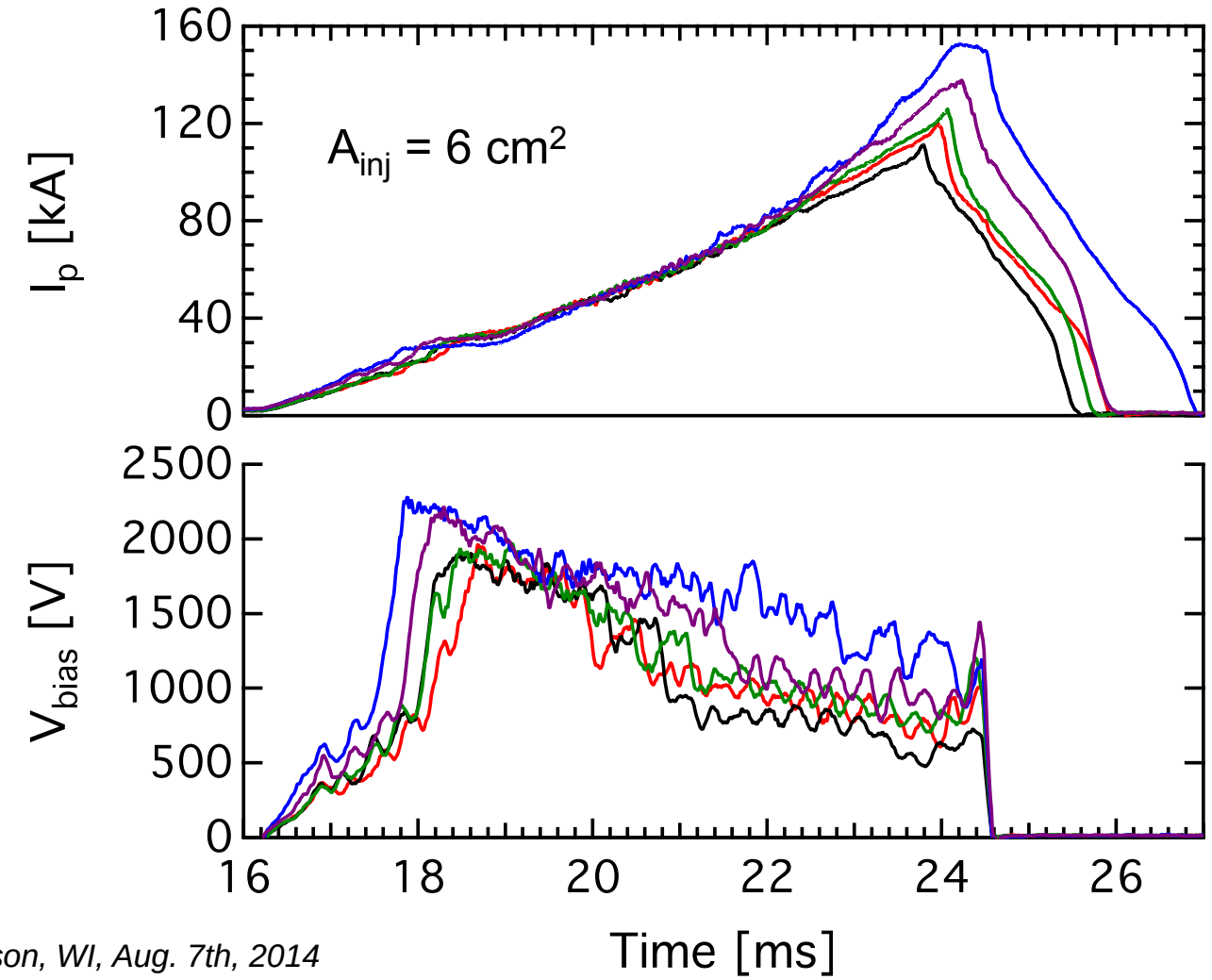
Inductive voltages include:

- **PF ramping**
  - Provides up to ~1 V inductive current drive during the current ramp
- **Geometry evolution**
  - Time-changing external inductance provides up to ~3 V of inductive surface voltage
- **Inductive reactance**
  - Plasma inductance demands up to ~ 4 V of additional surface voltage during the current ramp

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Initial Experimental Results Support I_p Scalability with Applied V_{eff}

- Gains in I_p trend with added V_{eff}
  - V_{inj} increased with incremental decreases in injector gas
  - Discharges with identical programming otherwise

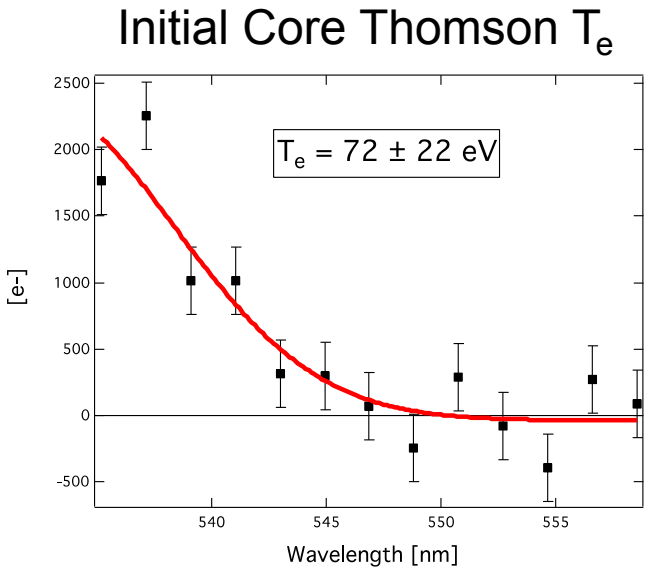


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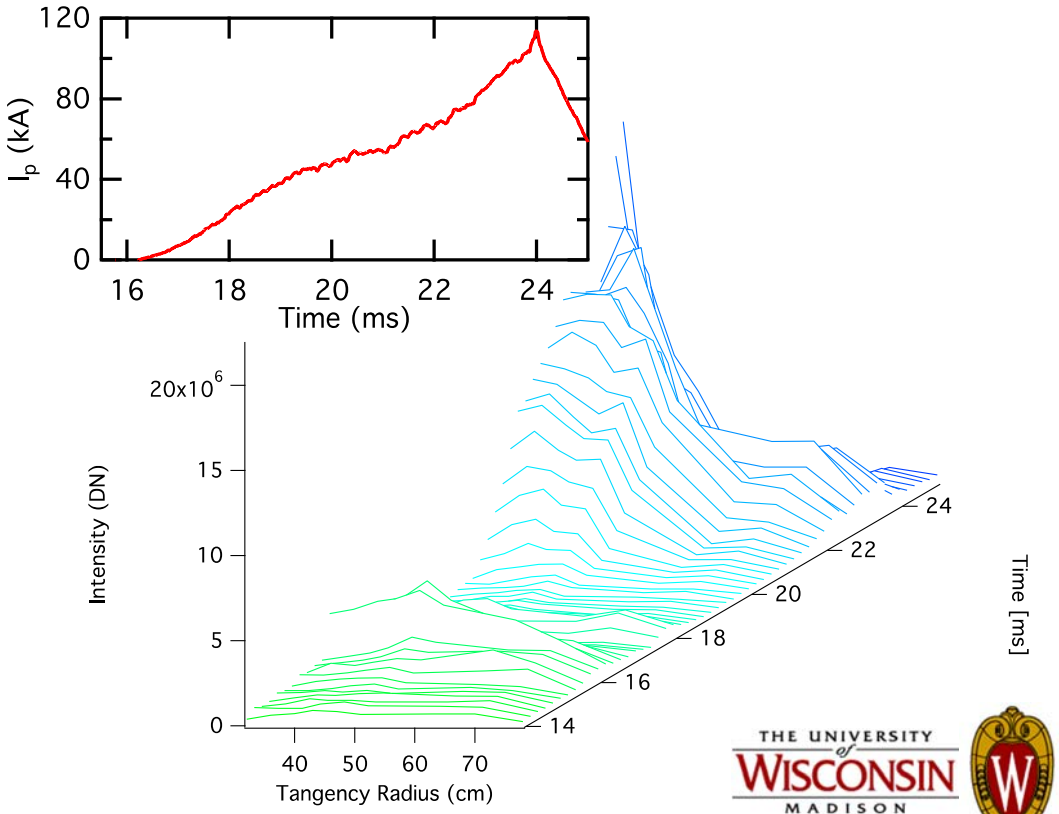
I_p Scaling with Device Size & V_{eff} Determined by Confinement, $T_e(R,t)$ Model

- Electron confinement, resistivity determine dissipation losses
  - Possibility of Ohmic, stochastic, and/or multi-zone confinement
  - Implications for startup scalability, required V_{eff}

- Thomson scattering profile measurements imminent



- O-V chordal profiles suggest:
  - T_e(core) > T_e(edge) as I_p grows
  - Flat profile during relaxation phase

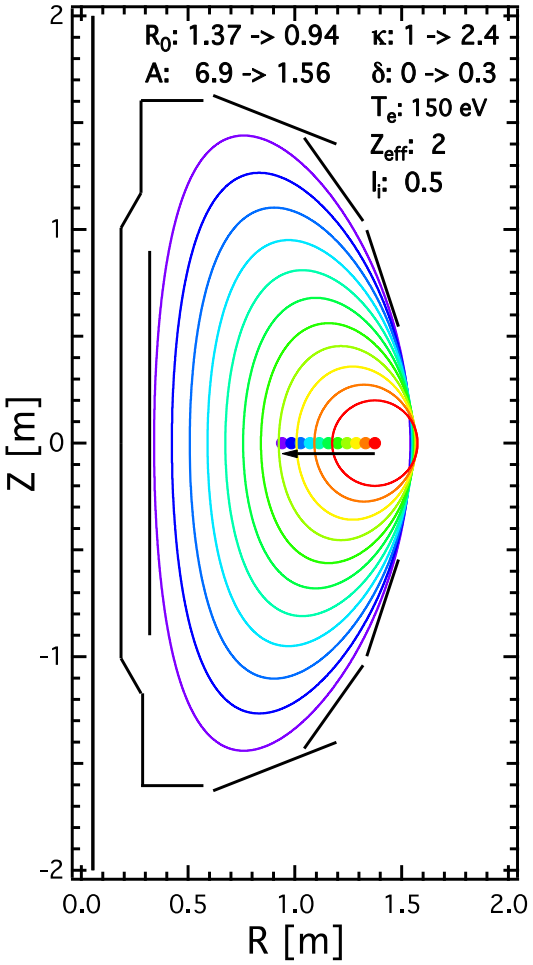


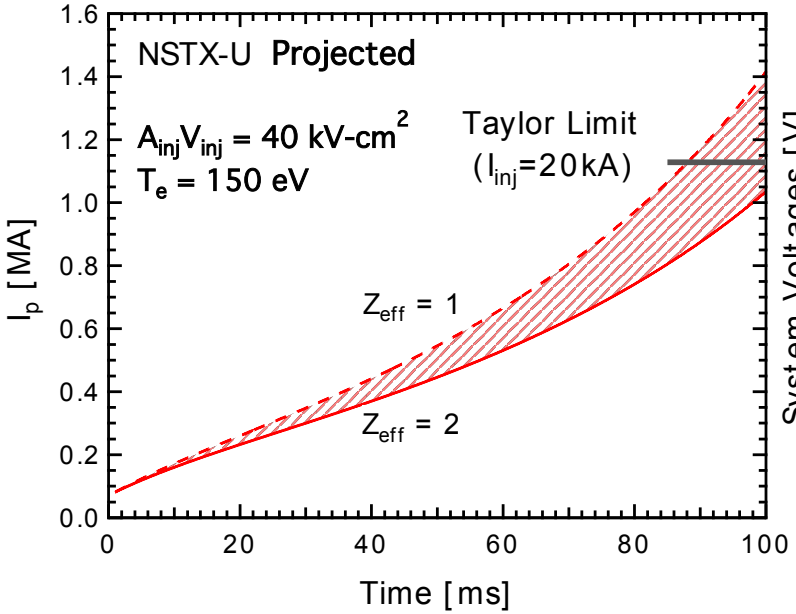
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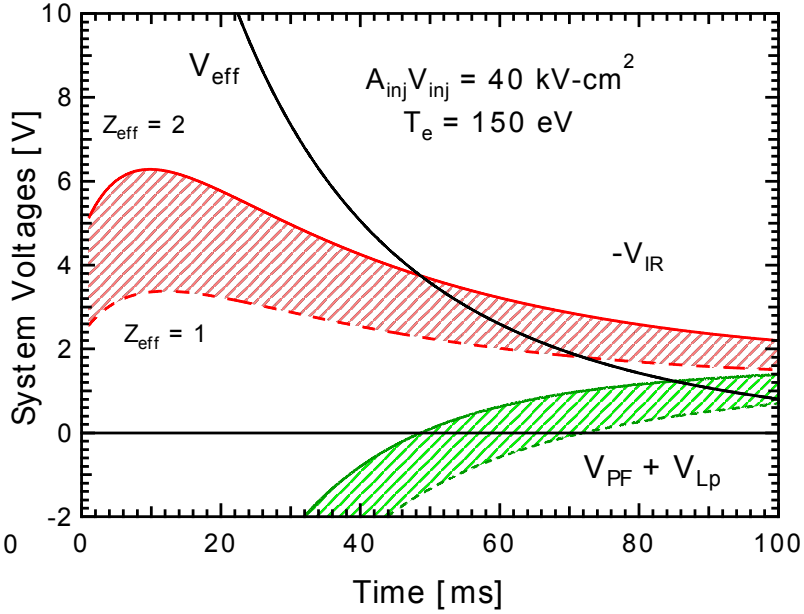


LHI Startup Scalability

Model Applied to NSTX-U Geometry for Initial I_p~1 MA Start-up Scenario Prediction







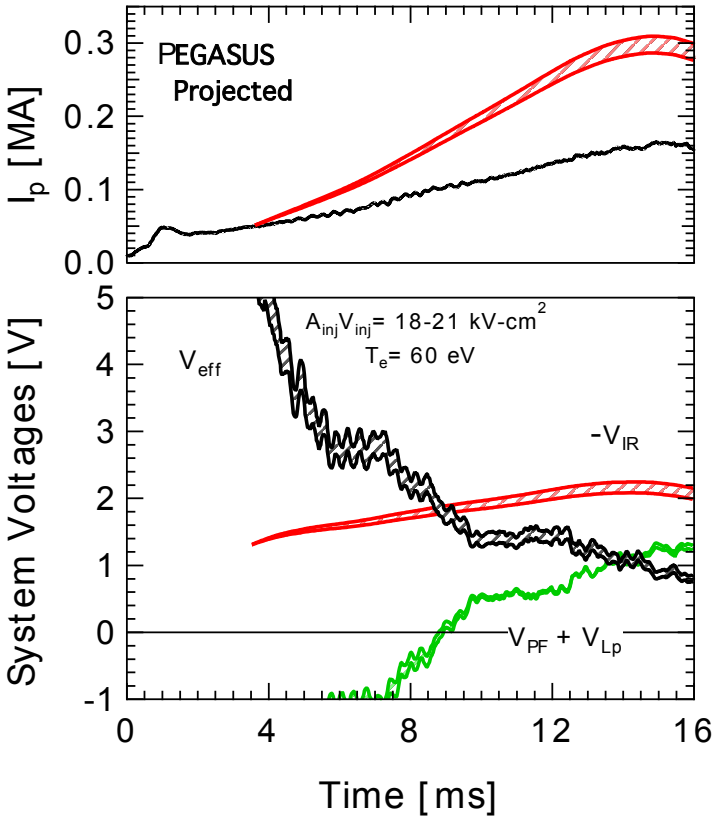
- Significant LHI drive required to achieve I_p ~ 1 MA
  - Predicted A_{inj} V_{inj} requirement estimates: ~ 40 cm^2-kV

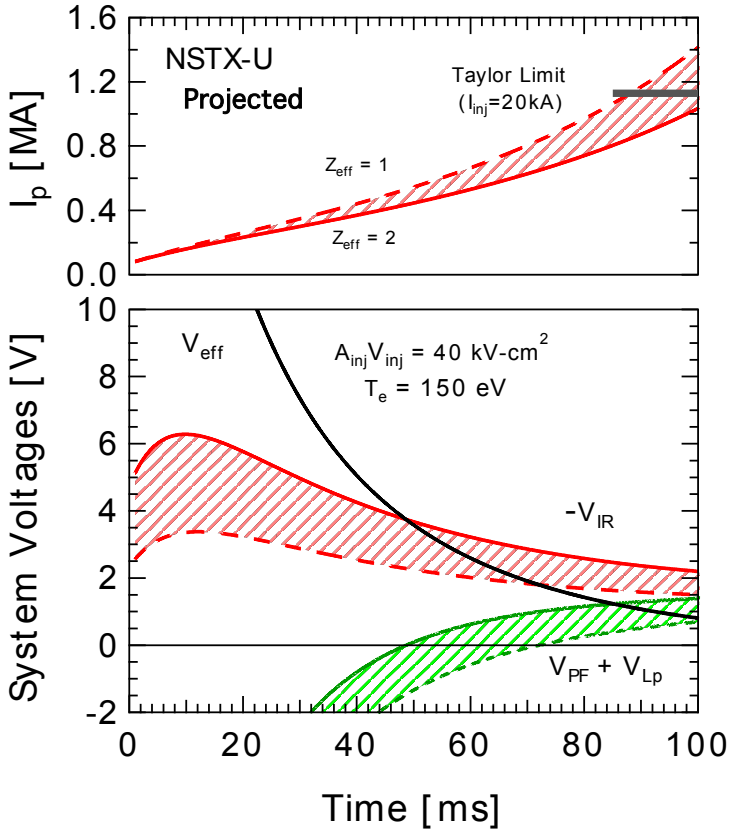
*C. Neumeyer *et all* (2009) *23rd IEEE/NPSS Symposium on Fusion Engineering*

**C. Neumeyer (2001) "NSTX Internal Hardware Dimensions"
http://nstx.pppl.gov/nstx/Engineering/NSTX_Eng_Site/Technical/Machine/NSTX_Engr_Machine_Dims_cm.html

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$I_p \geq 300$ kA on Pegasus in LHI-Drive Dominated Regime





- Modeling indicates Pegasus LHI discharges at I_p ~ 0.3 MA enter LHI-CD dominant regime
  - Expected at $A_{inj}V_{inj} \sim 20$ cm^2-kV
  - $I_{inj} = 20$ kA for sufficient Taylor Limit
- Additional issues for larger machines:
  - Injector heat loads at long-pulse
  - Possible B_T dependence for injector aperture

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Upgraded Injectors, TF Power-Supplies Enable LHI-CD Dominated Regime Ops

- Injector upgrades enable high V_{eff}:
  - Additional injector arrays, geometries
  - Injector impedance control through:
    - High-voltage standoff capability
    - Passive cathode-spot protection
    - Independent power-supplies
    - High-speed injector-gas control
    - Low injector-gas arc initializer system

$$V_{eff} \sim A_{inj} V_{bias}$$

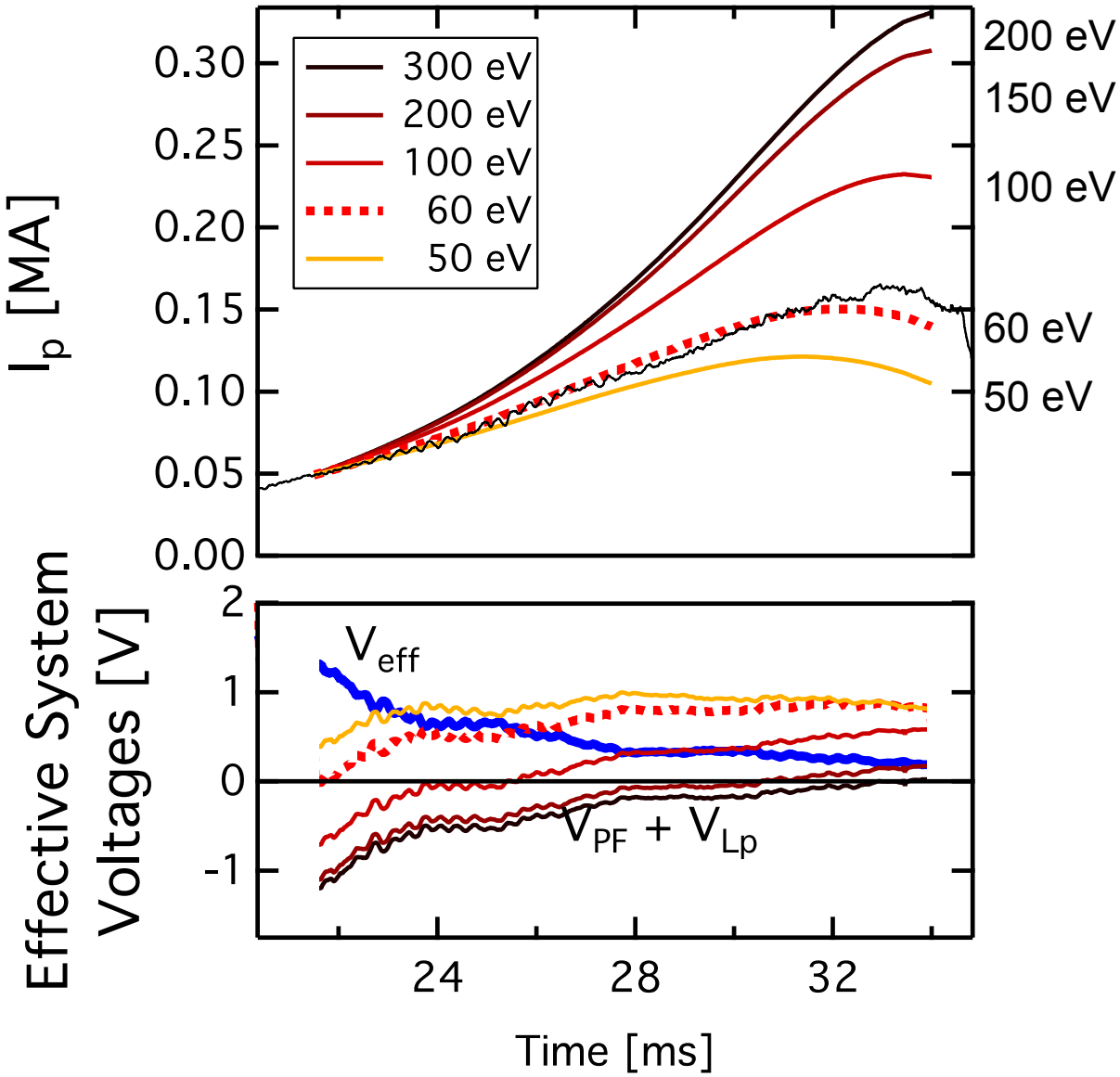


- 2x TF Field upgrade for 1.4x increase in Taylor Limit
  - Enables higher max I_p
  - Allows higher I_p(t=relaxation), the initial condition for power-balance evolution

Improved Confinement Offers an Additional Path to the LHI-Dominated Regime

- Decreased resistivity permits higher $\frac{\partial}{\partial t} I_p$
  - Same V_{eff} attains higher I_p
- Increased confinement gives second pathway to the LHI-dominant regime
  - Still attained at $I_p \sim 0.3$ MA
  - Decreased injector requirements:

    $A_{inj} V_{inj} \sim 8$ cm^2-kV
    with $T_e \sim 150$ - 200eV



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Summary

- Outboard LHI startup discharges undergo a dynamic geometric, inductive evolution
  - Early $I_p(t)$ reaches, subject to a Taylor relaxation current limit
  - Subsequent $I_p(t)$ evolution subject to strong geometric and inductive effects, power-balance
- A lumped-parameter, power-balance model is used to account for these dynamics, predict $I_p(t)$
  - Model is being experimentally tested, calibrated on Pegasus
- High performance startup on NSTX-U, Pegasus requires operations in a dominantly LHI driven regime
  - Confinement scaling into this regime important for startup scalability
- Injector, facility upgrades enable testing of the LHI drive-dominant regime on Pegasus

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